61A Extra Lecture 1

Announcements

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- Permanent lecture times are TBD, but probably Wednesday evening or Monday evening

Newton's Method

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!

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$f(x) = x^2 - 2$

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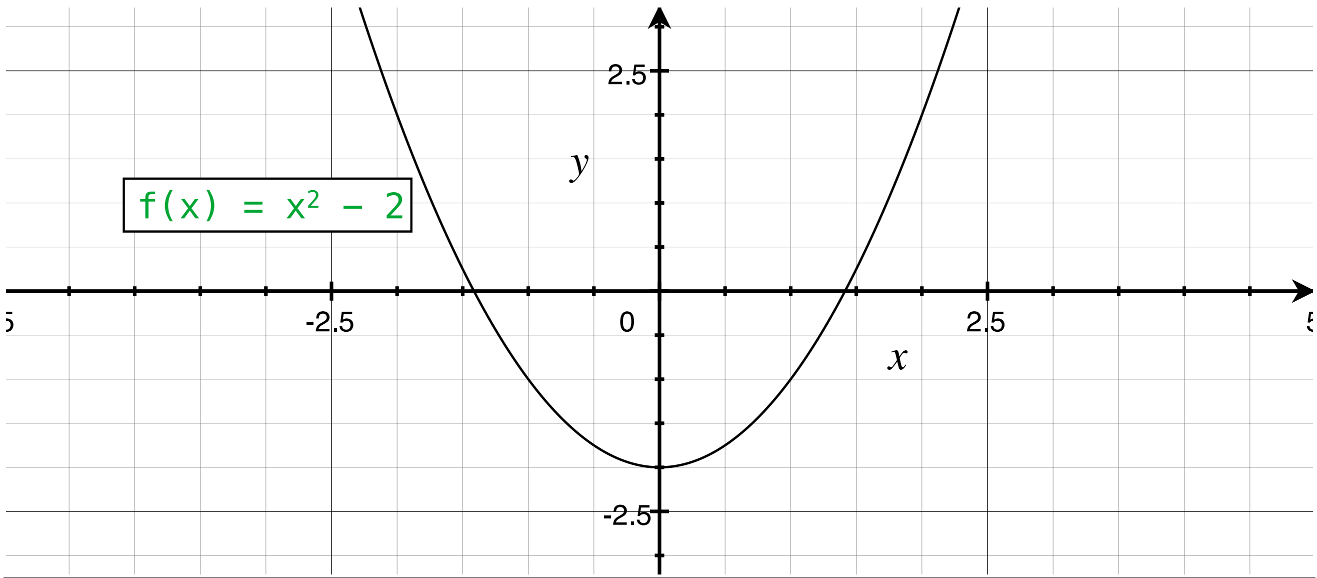

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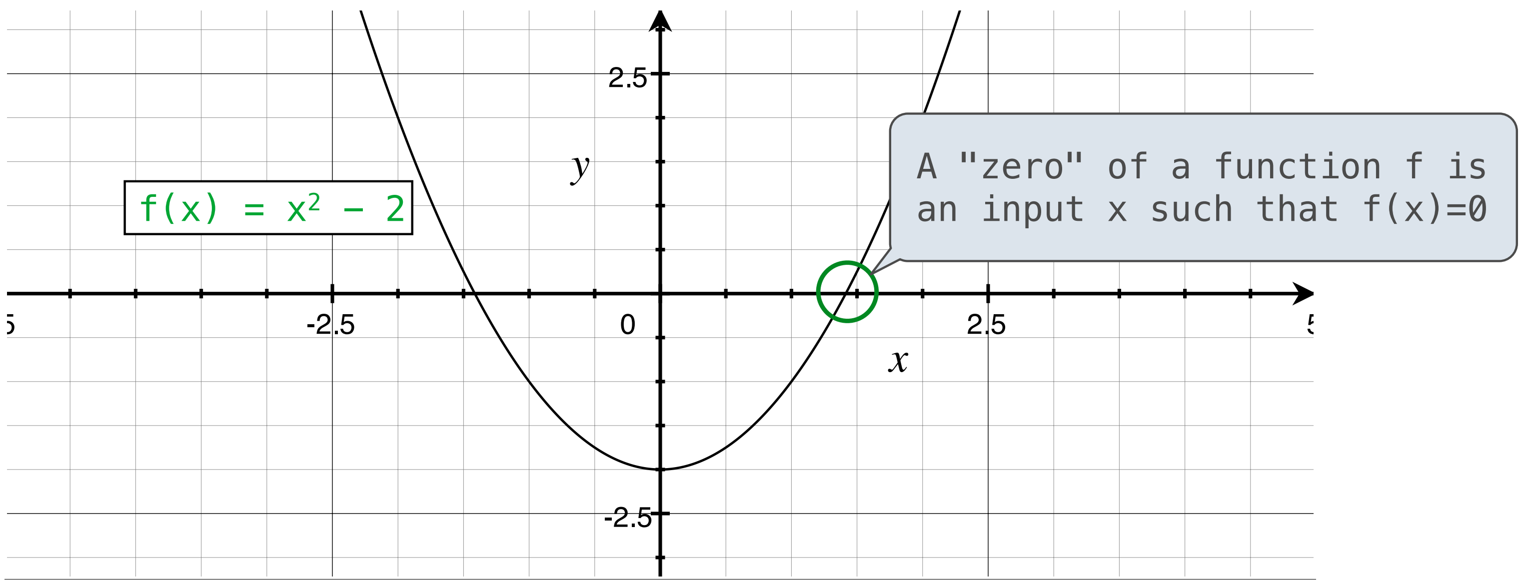

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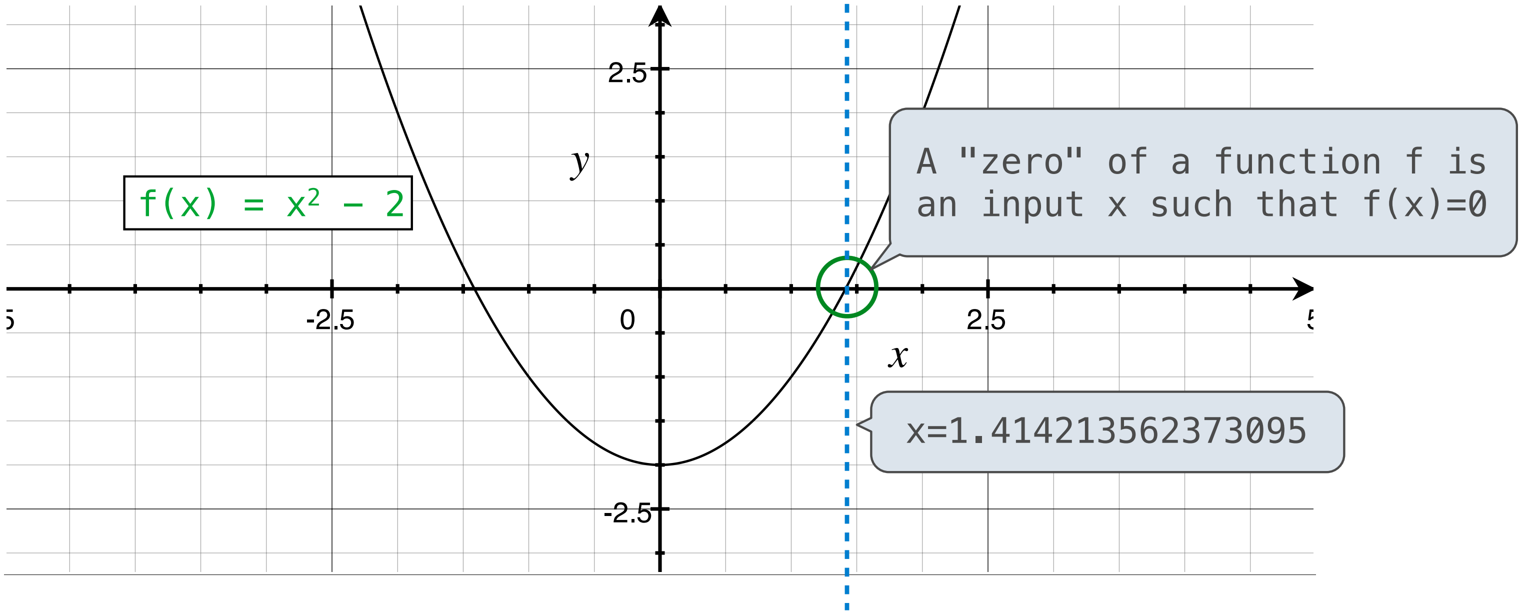

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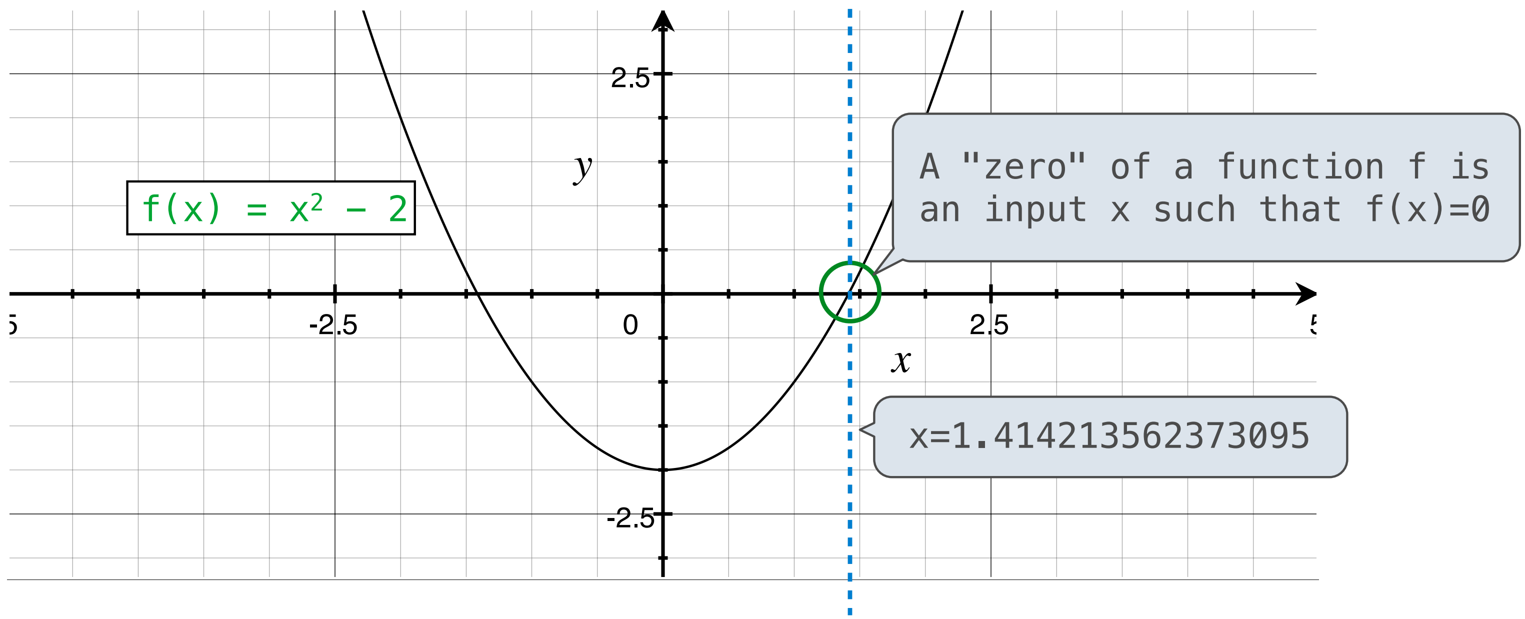

`Application: a method for computing square roots, cube roots, etc.`

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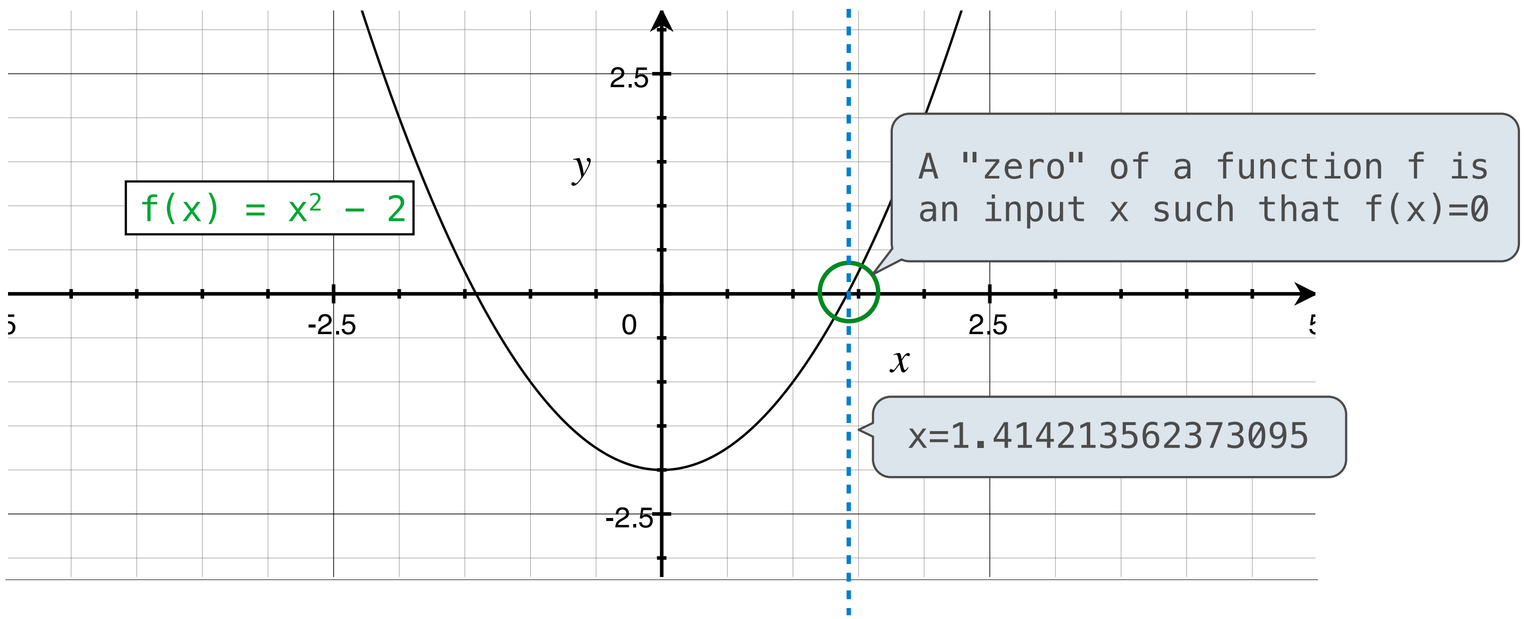

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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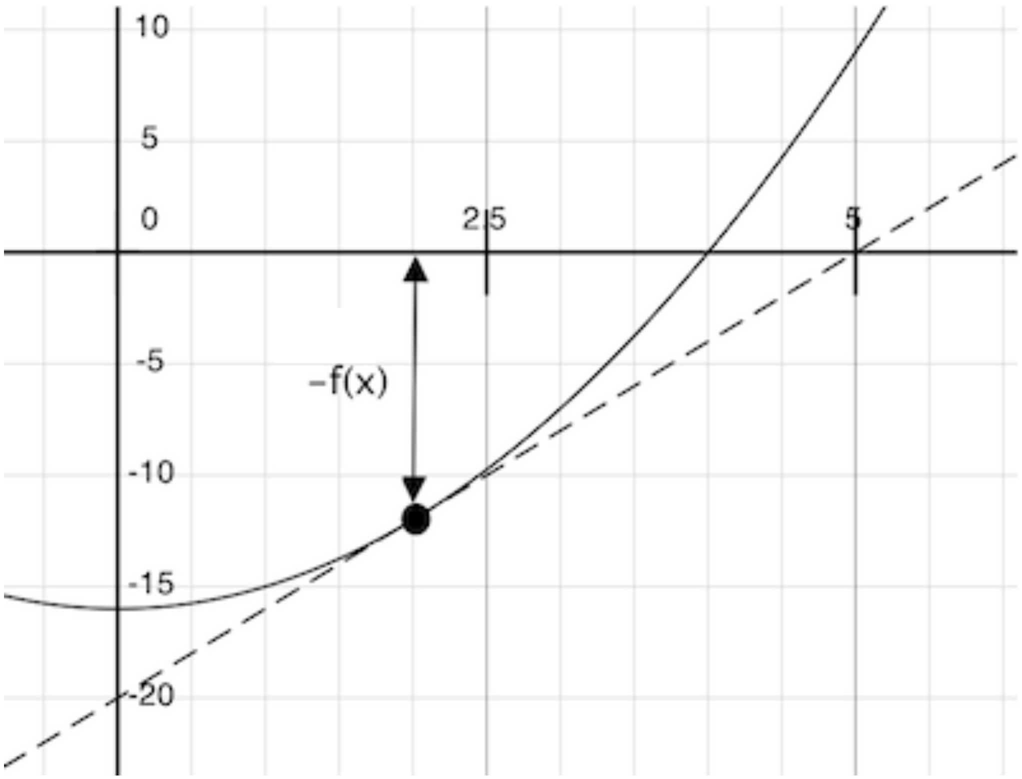

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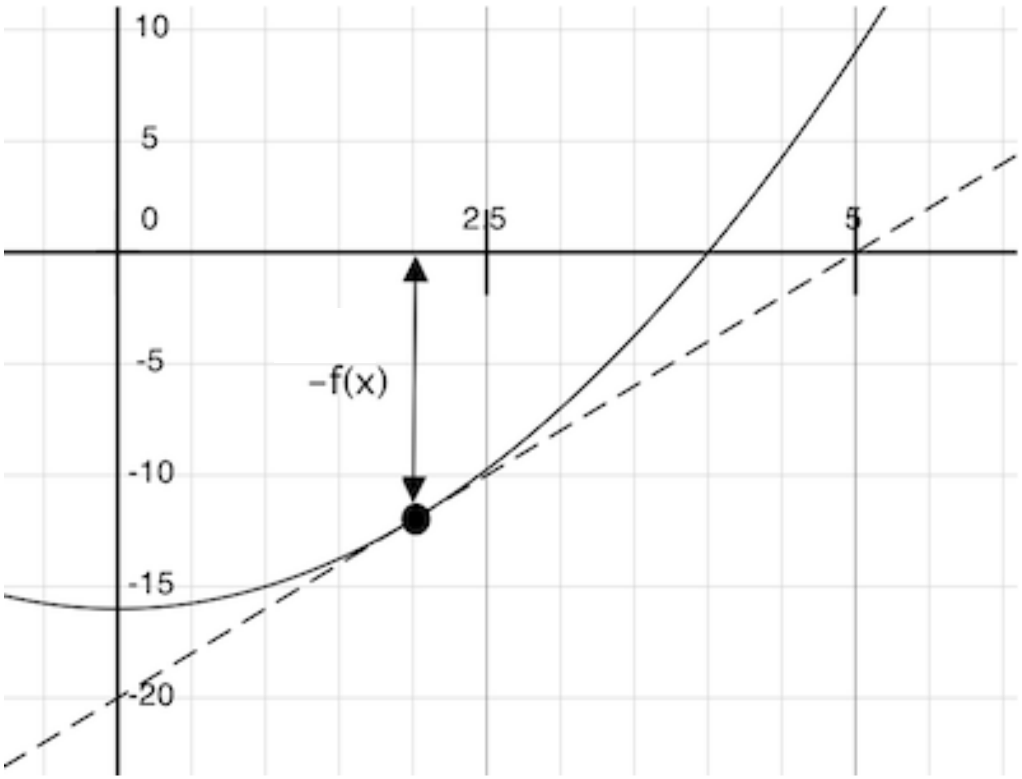

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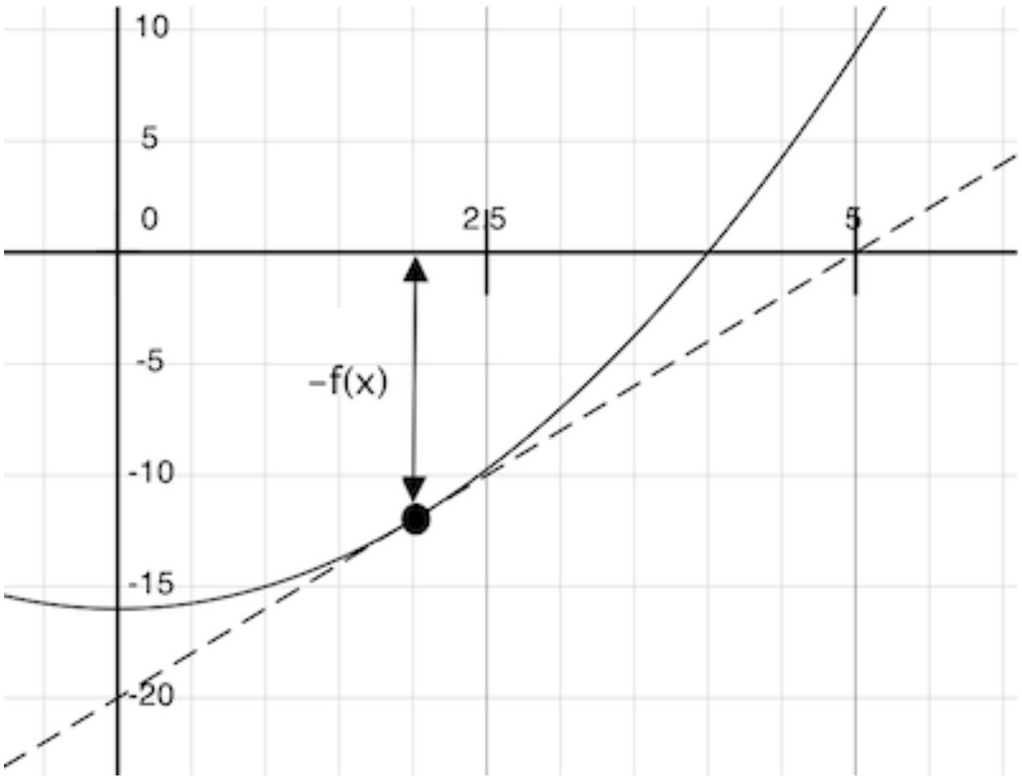

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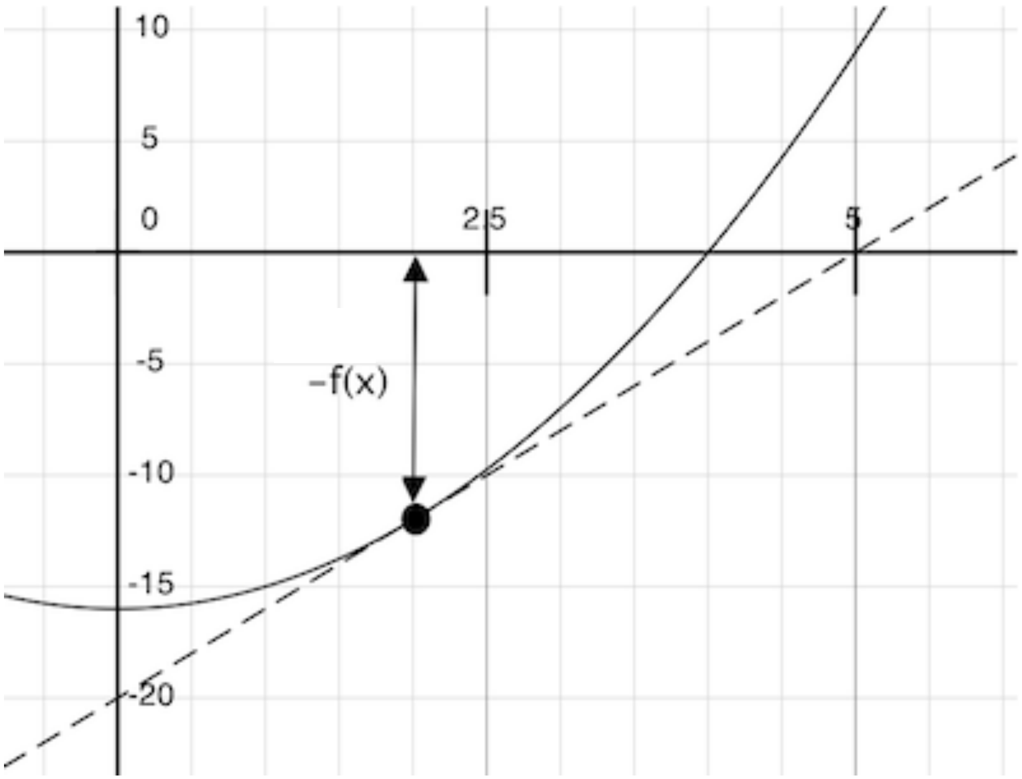
$$x - \frac{f(x)}{f'(x)}$$



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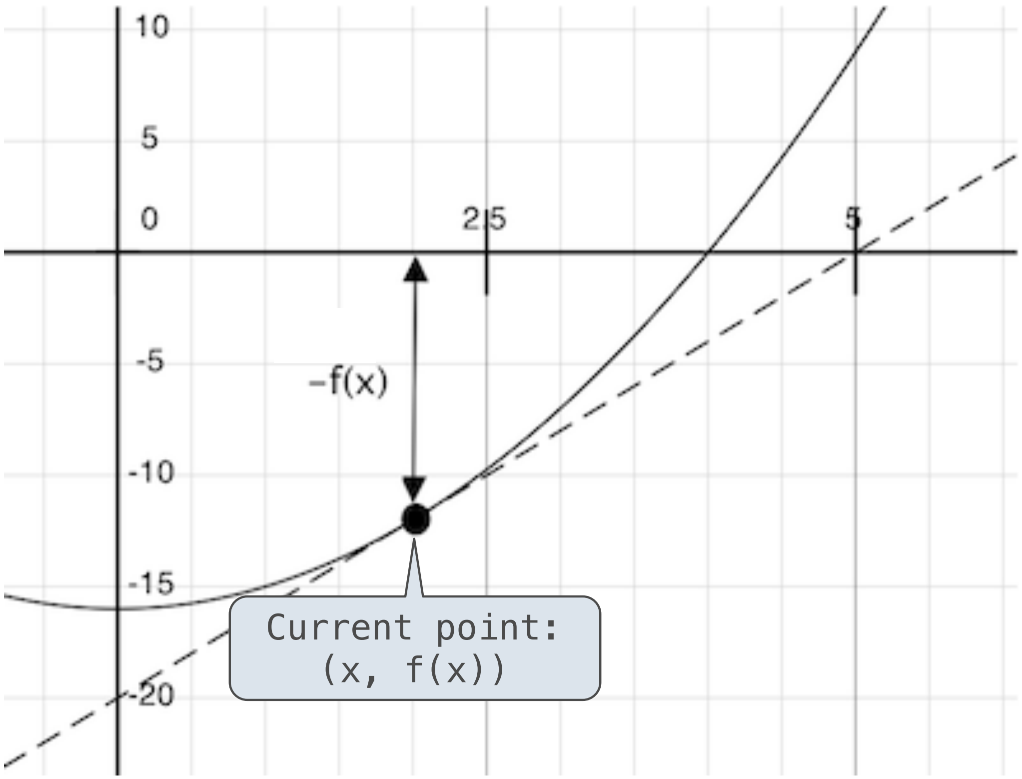

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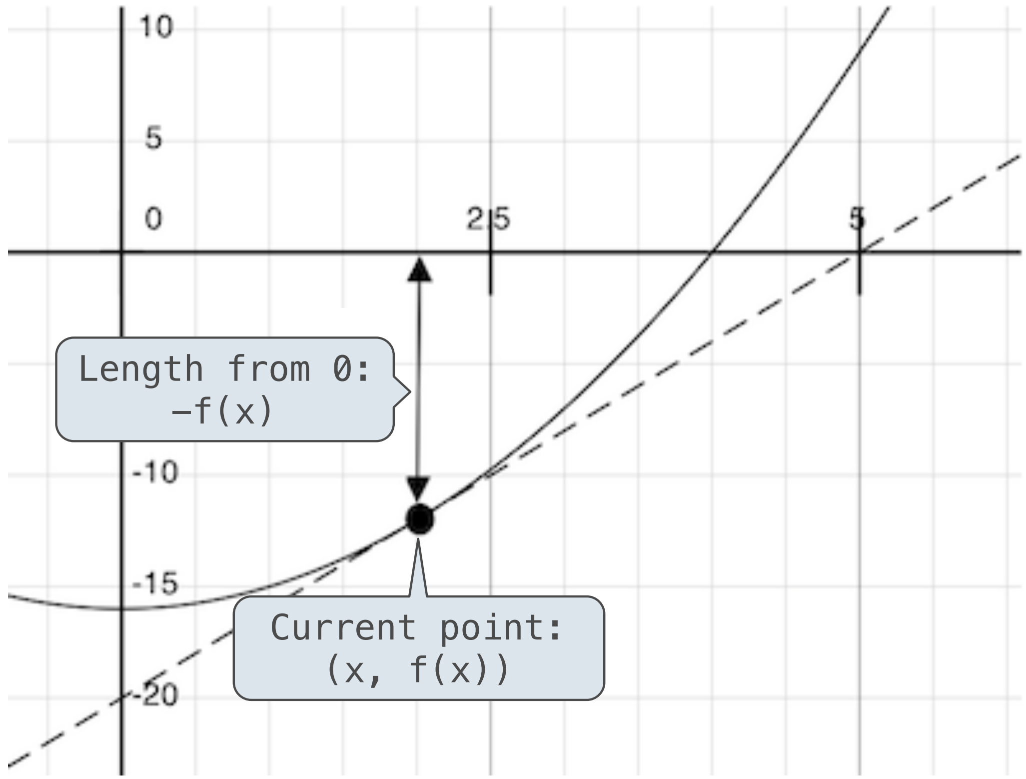

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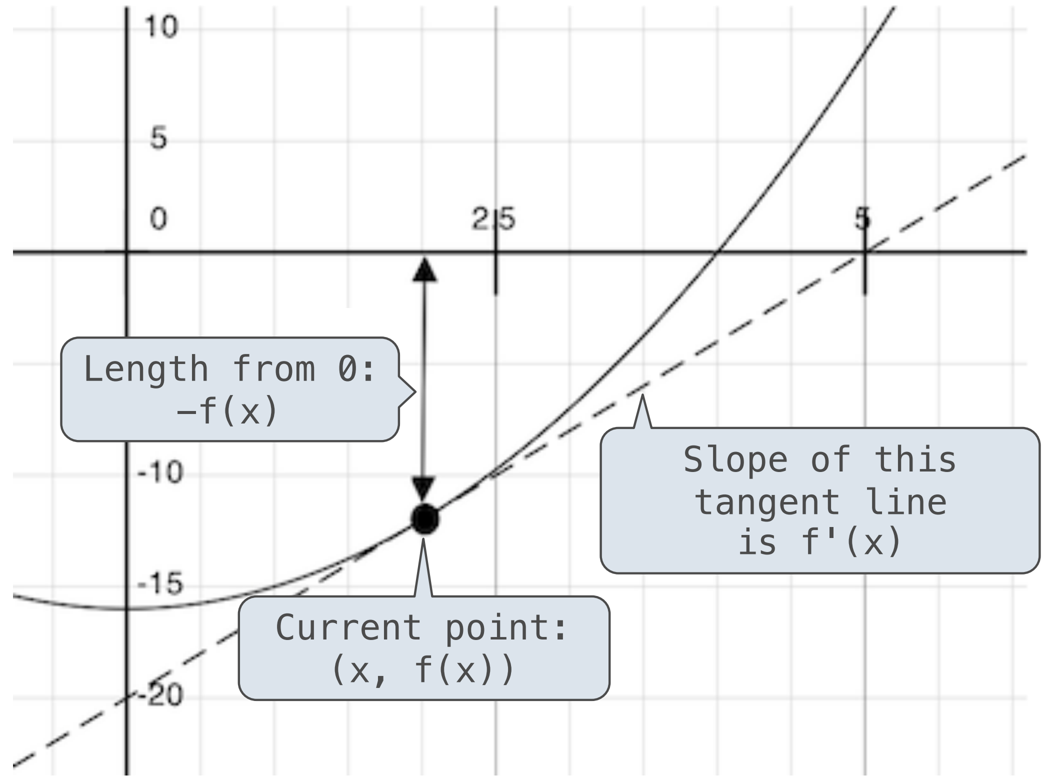

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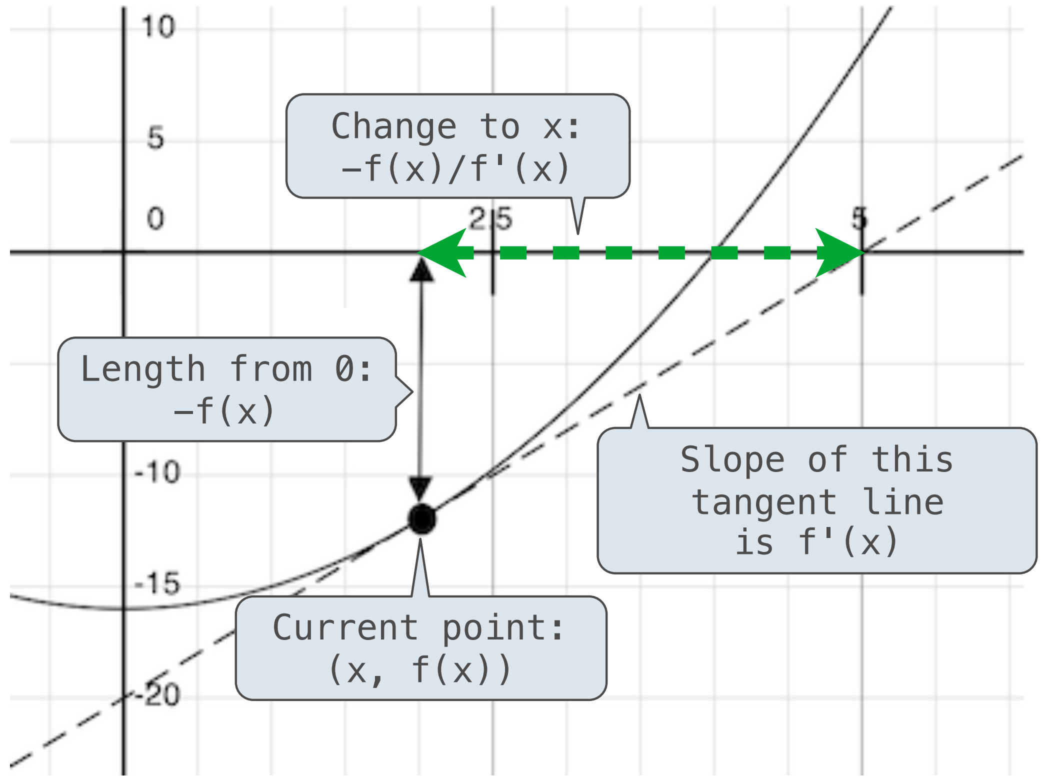

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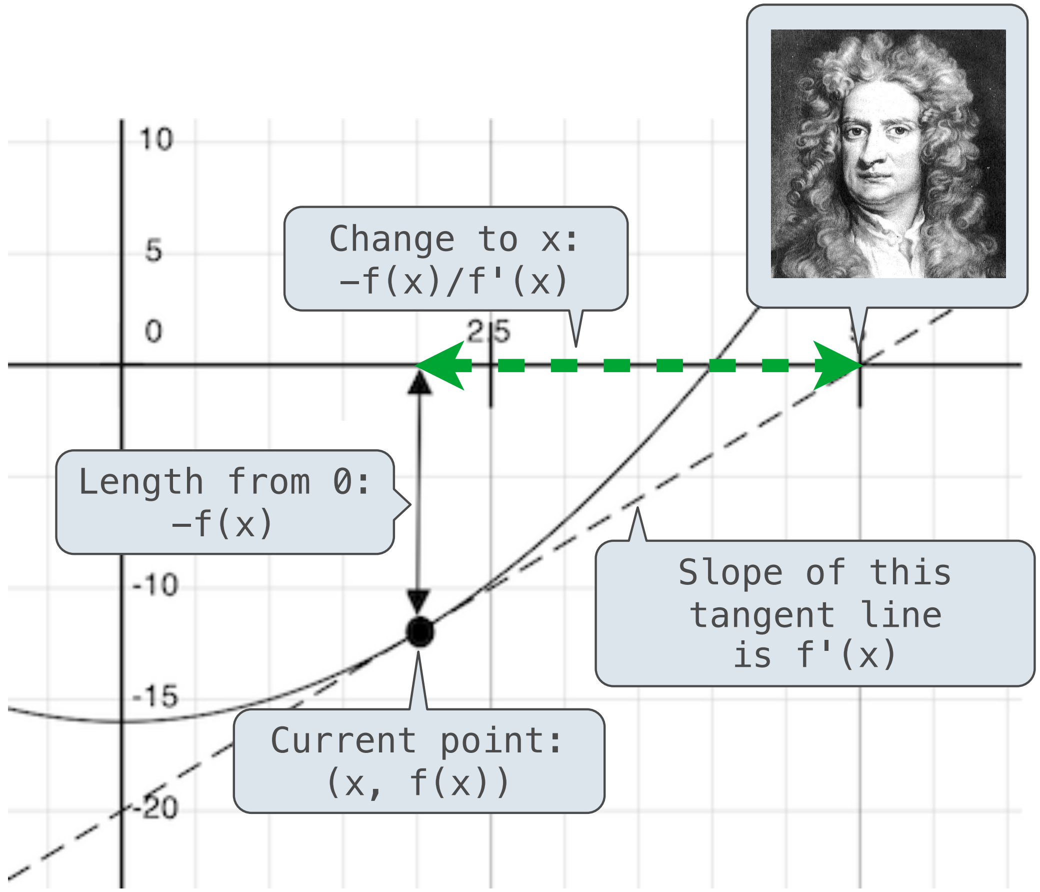

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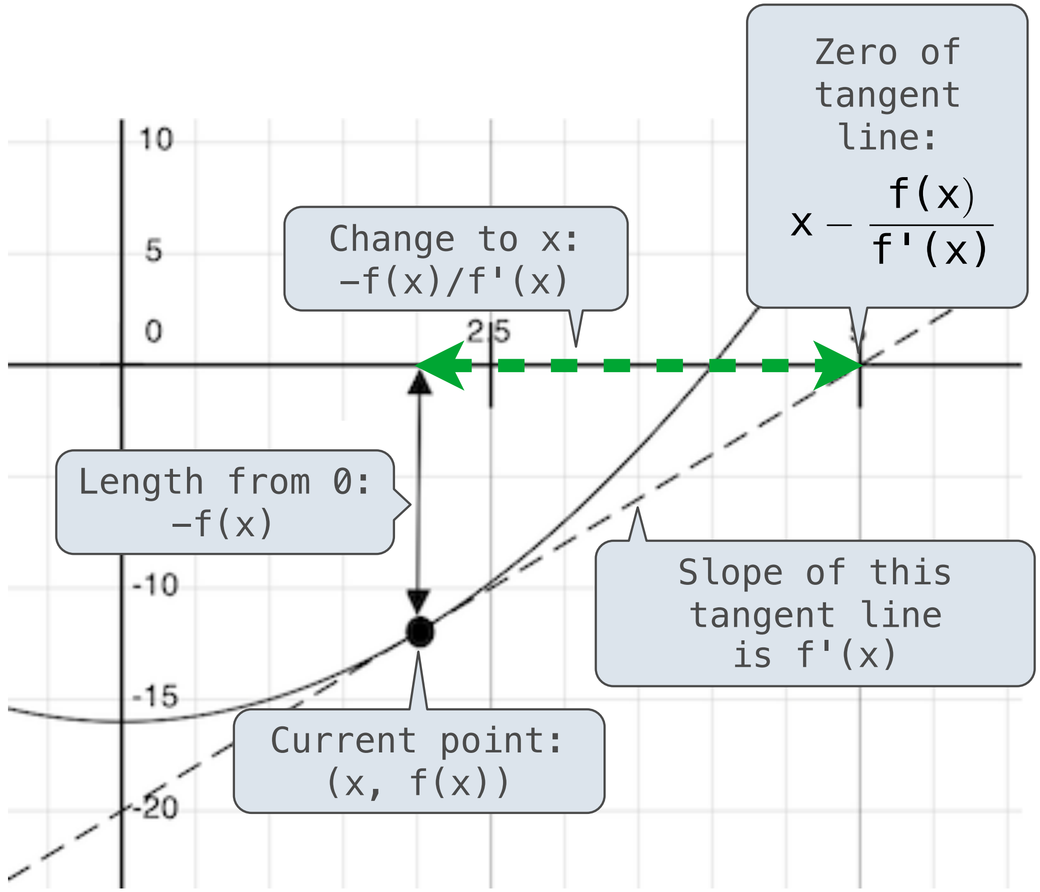

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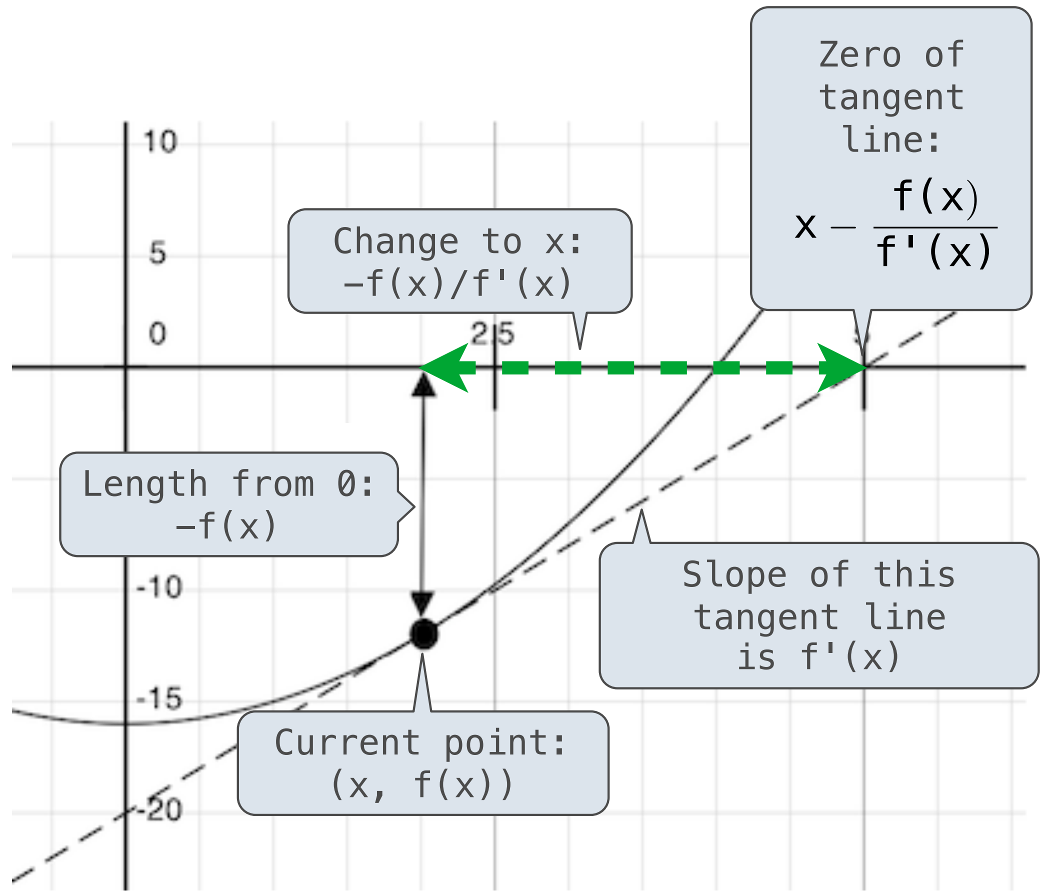

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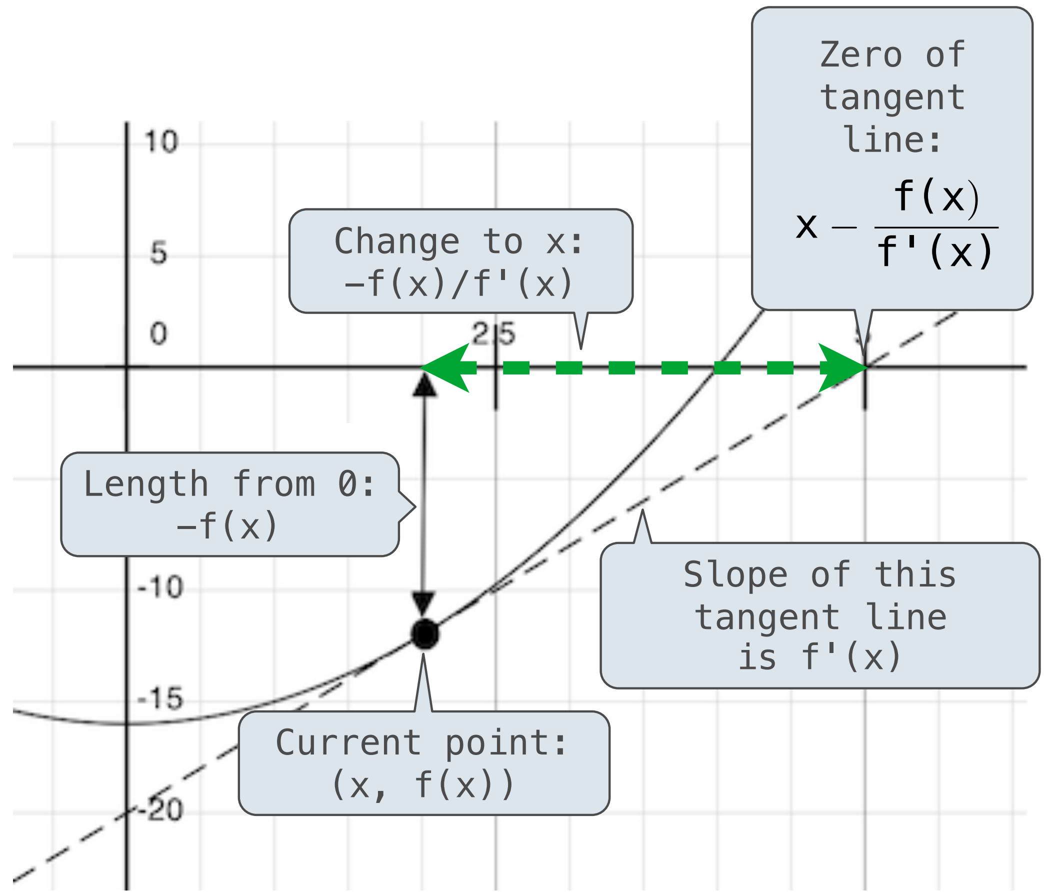

Using Newton's Method

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How to find the square root of 2?

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>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
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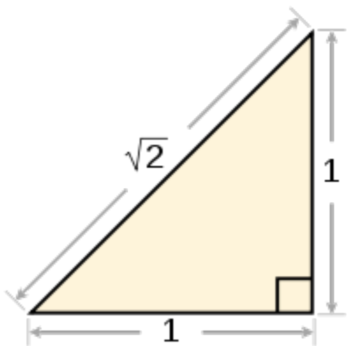


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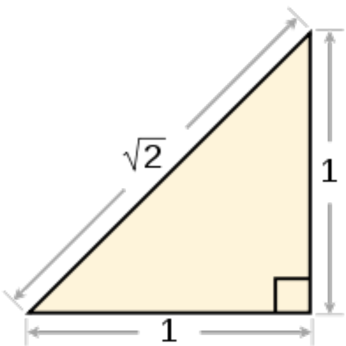


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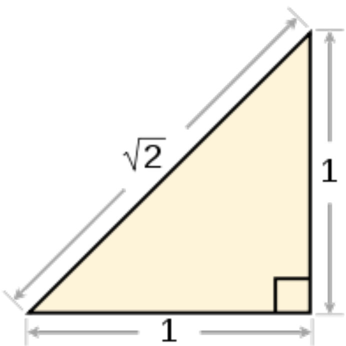


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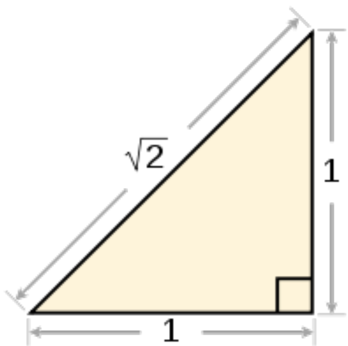


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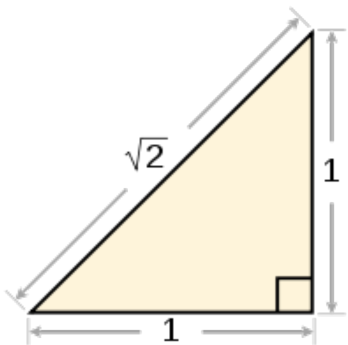


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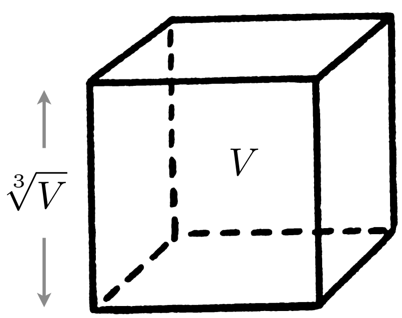

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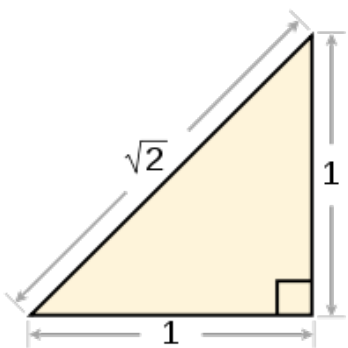


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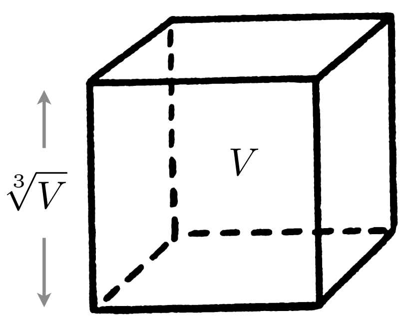


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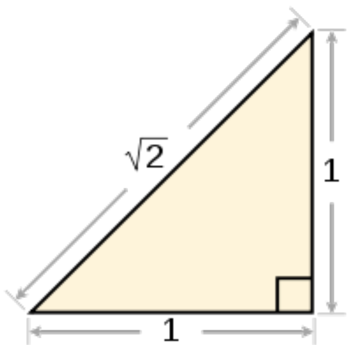


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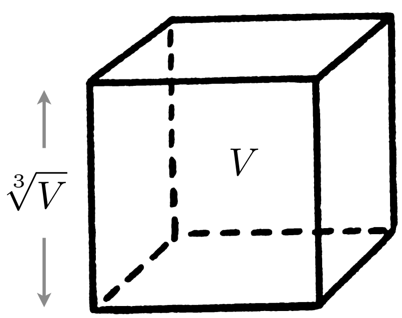


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Iterative Improvement

Special Case: Square Roots

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(Demo)

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Implementing Newton's Method

(Demo)

Extensions

Approximate Differentiation

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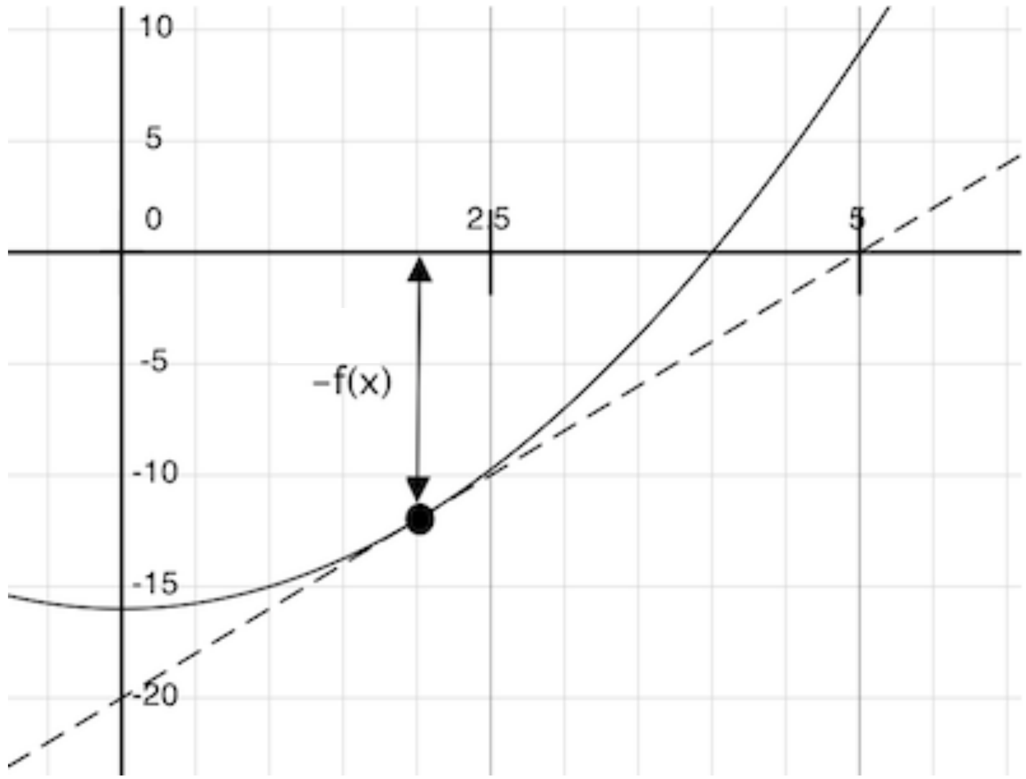
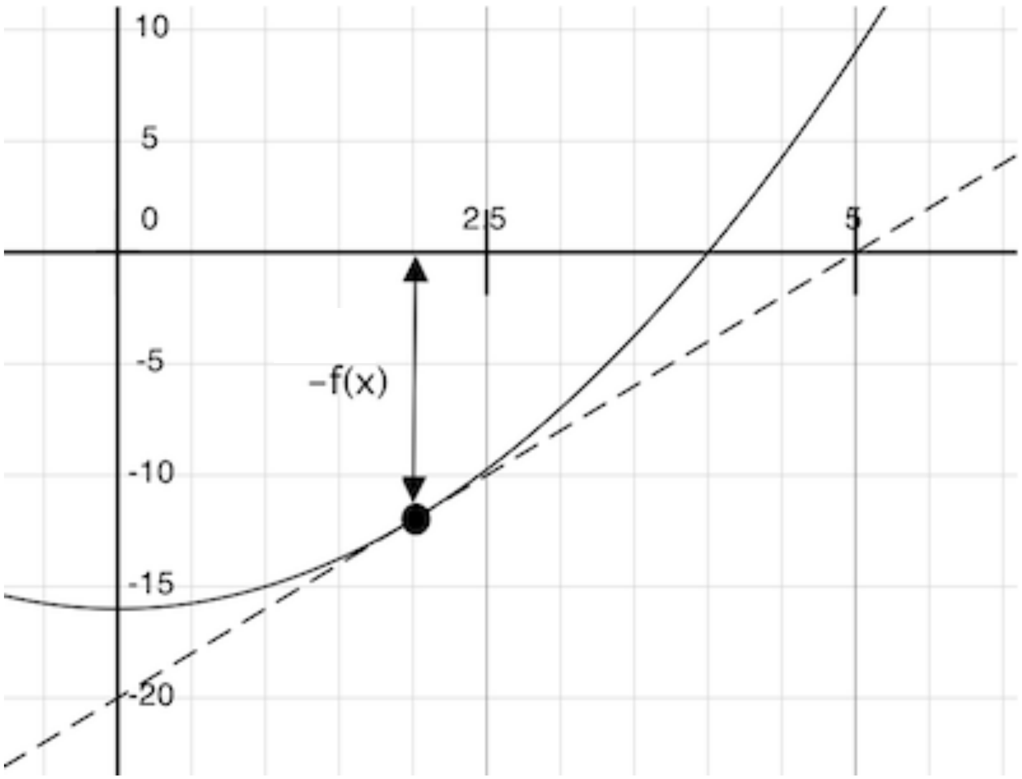


Approximate Differentiation

Differentiation can be performed symbolically or numerically



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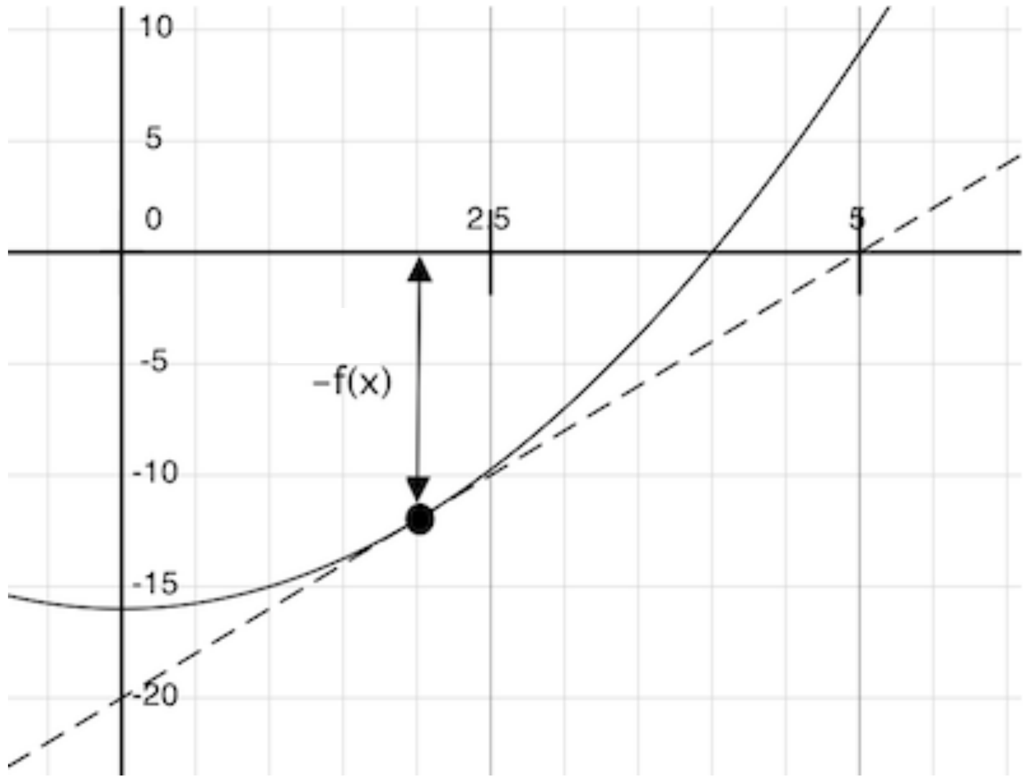
Differentiation can be performed symbolically or numerically

$f(x) = x^2 - 16$



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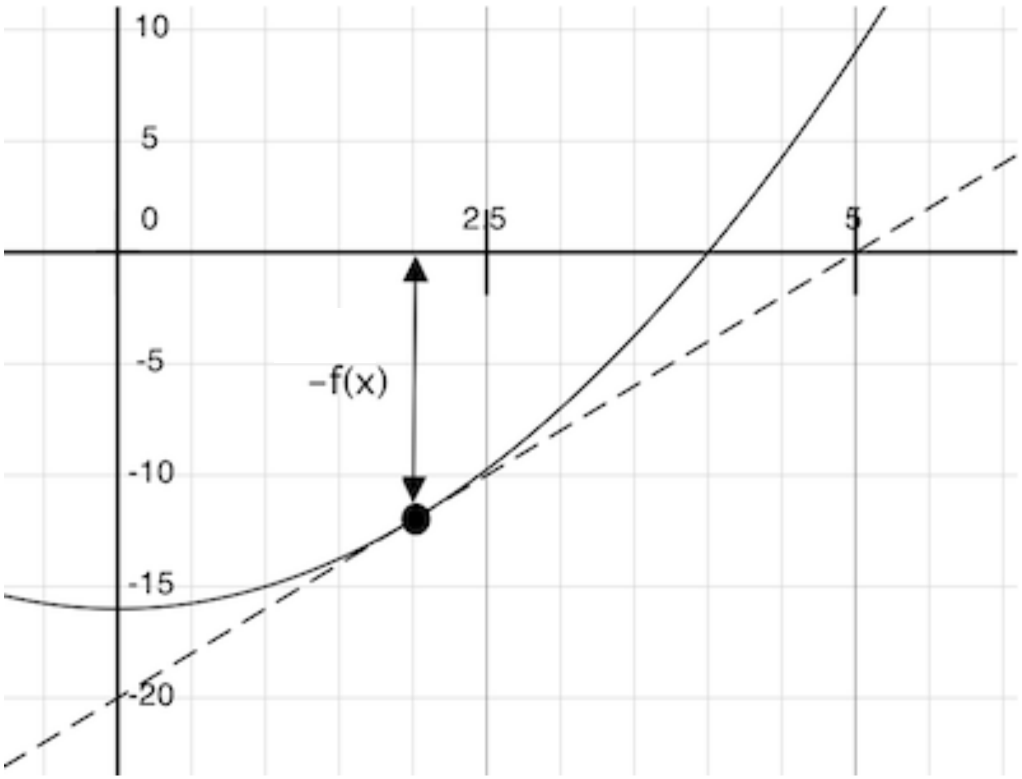

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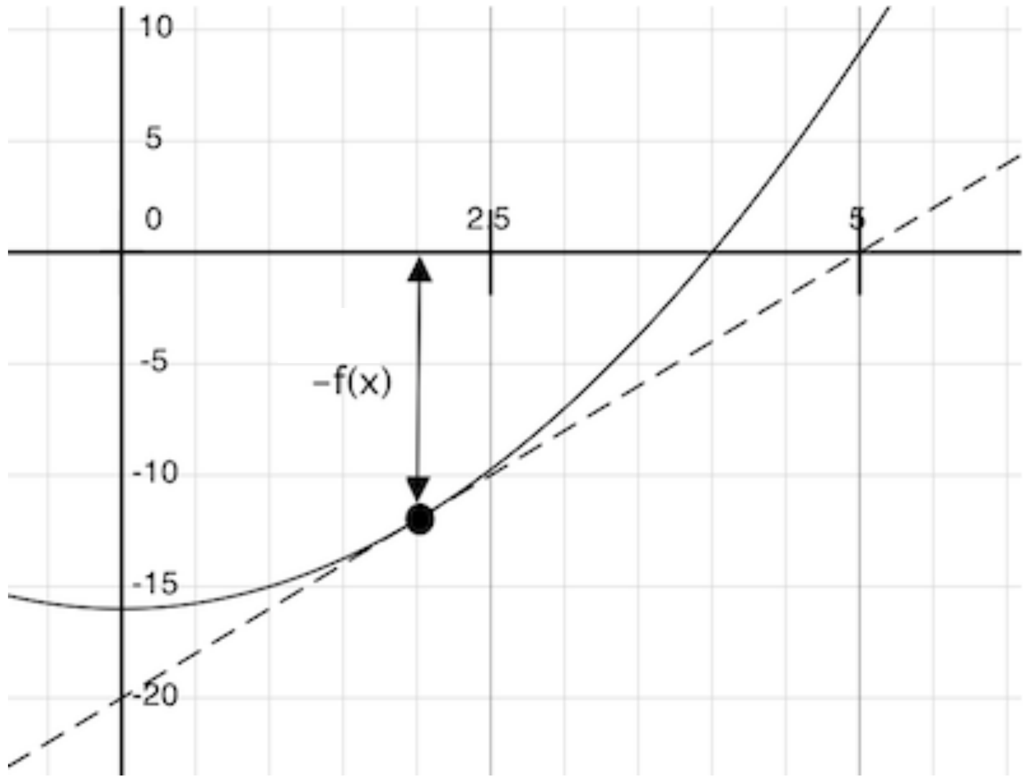

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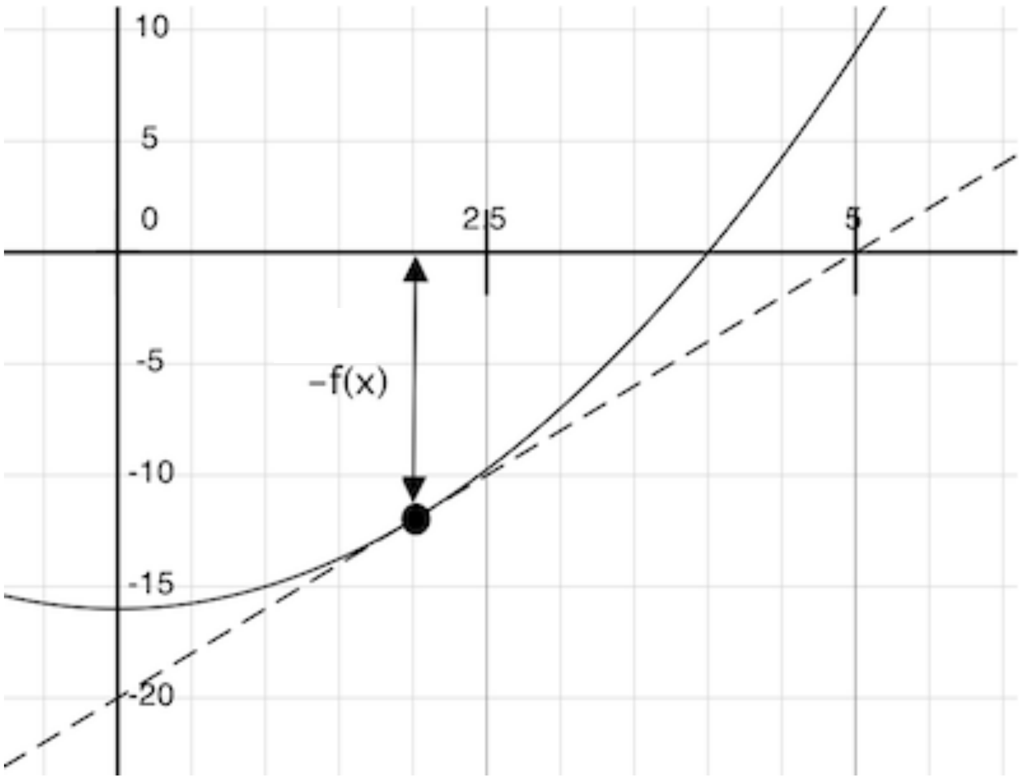

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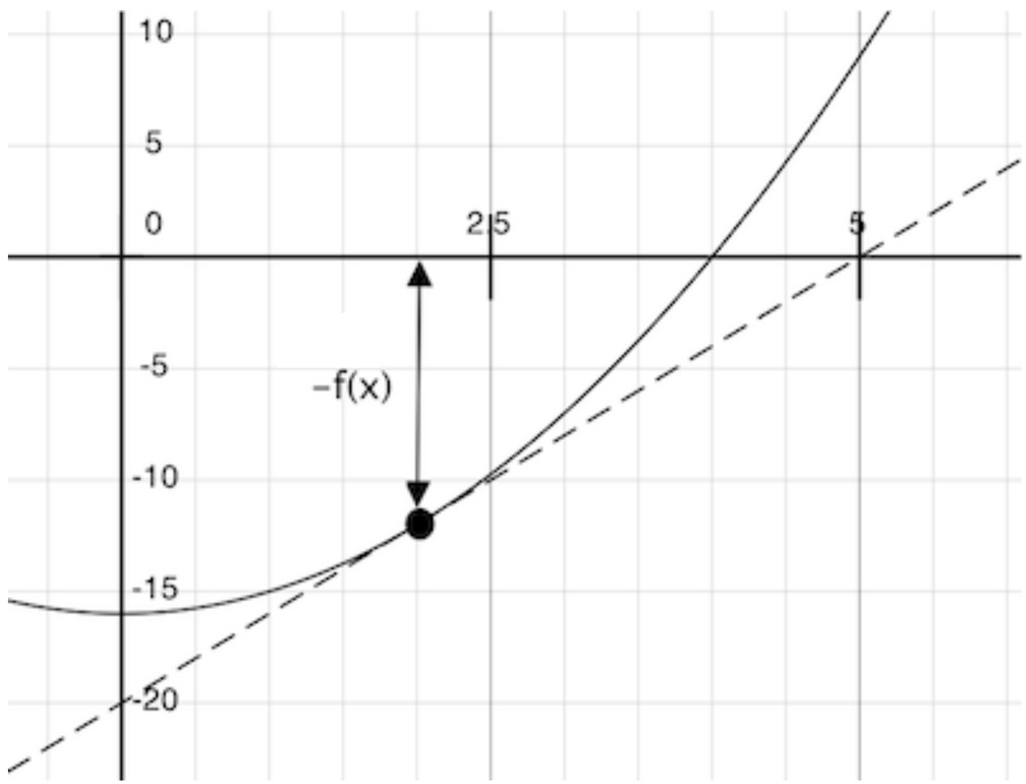

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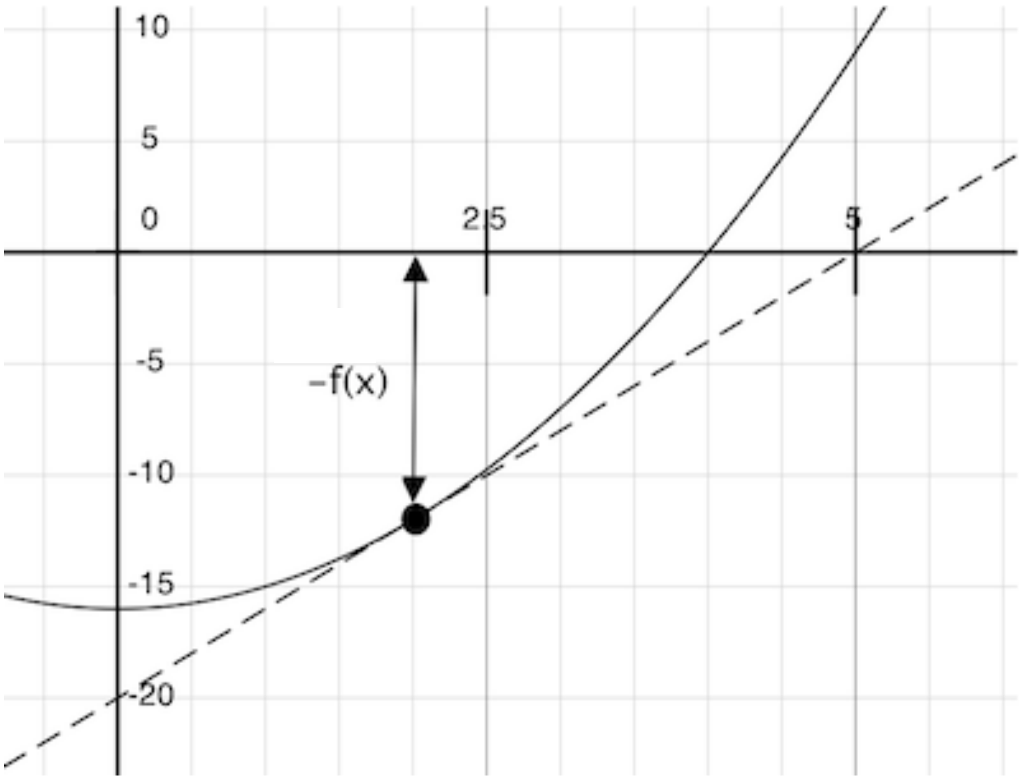

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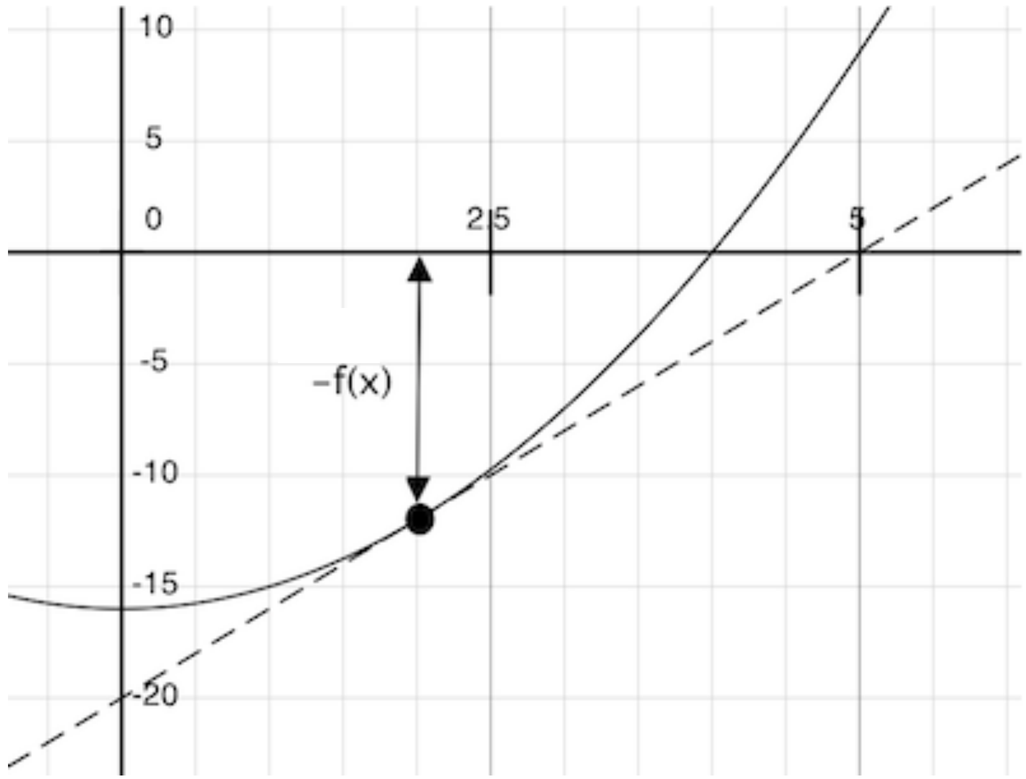

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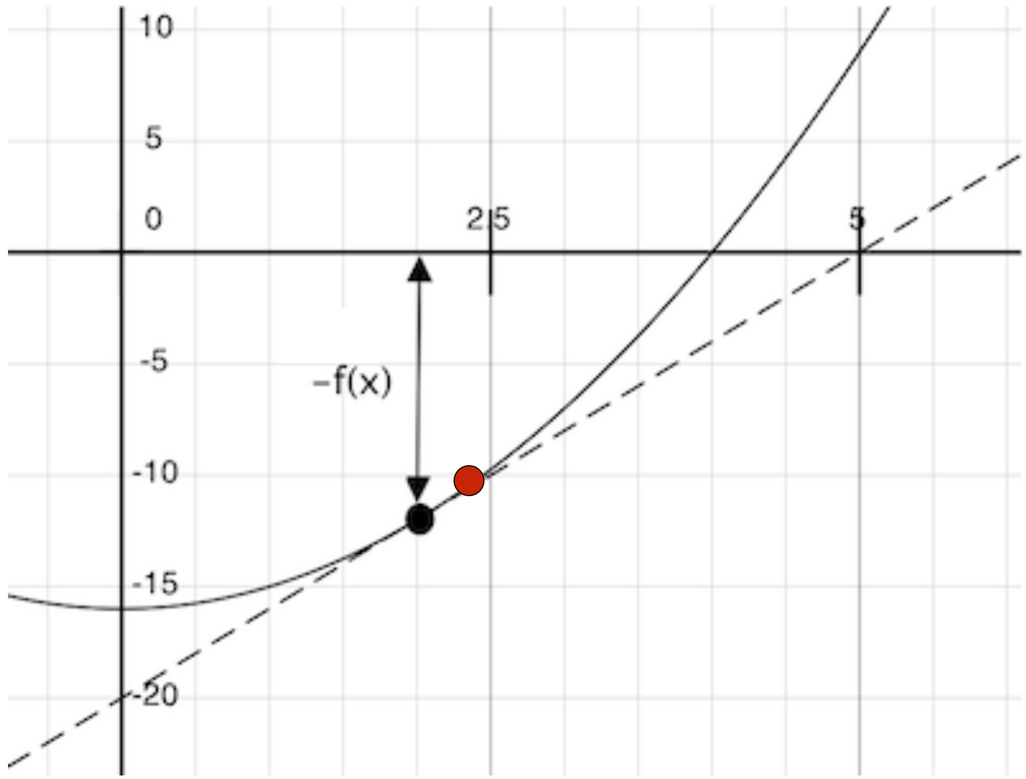

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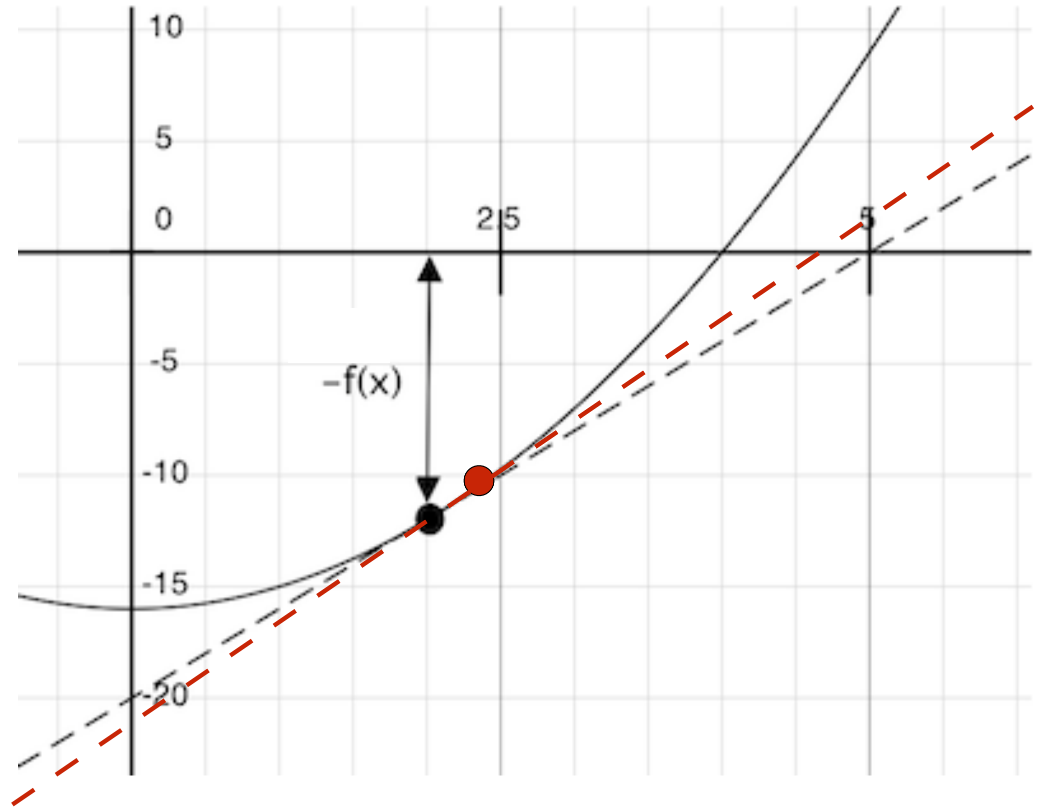

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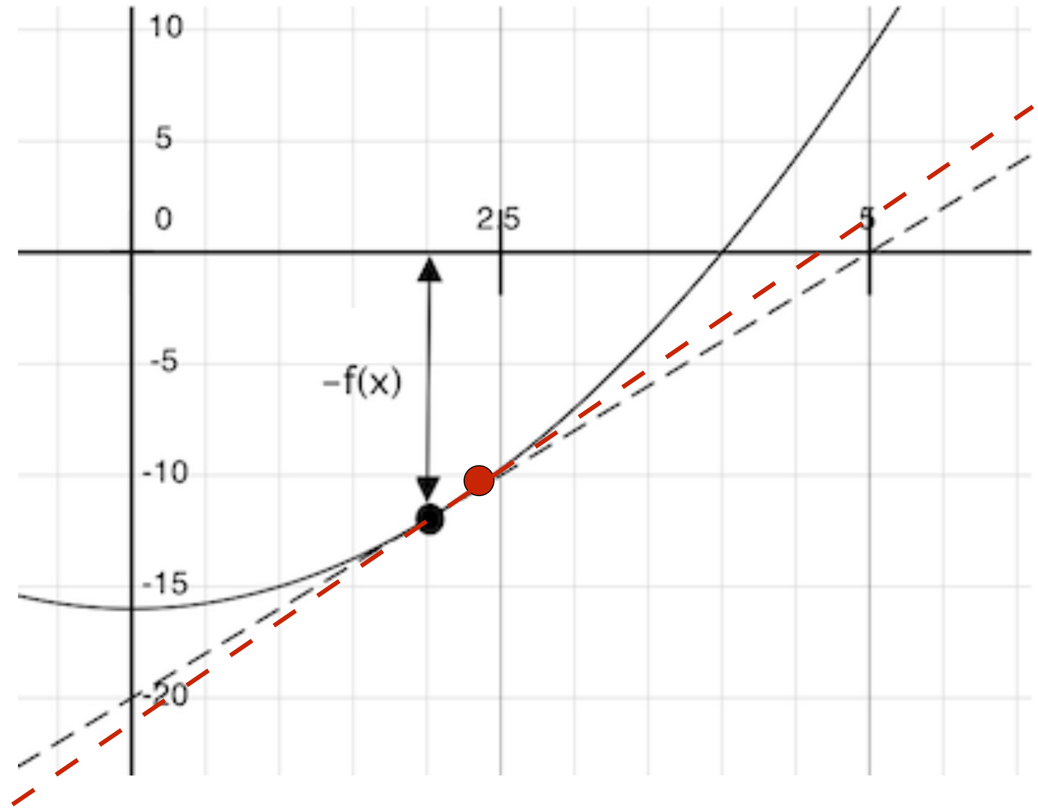
$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$ (if a is small)

(Demo)



Critical Points and Inverses

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Maxima, minima, and inflection points of a differentiable function occur
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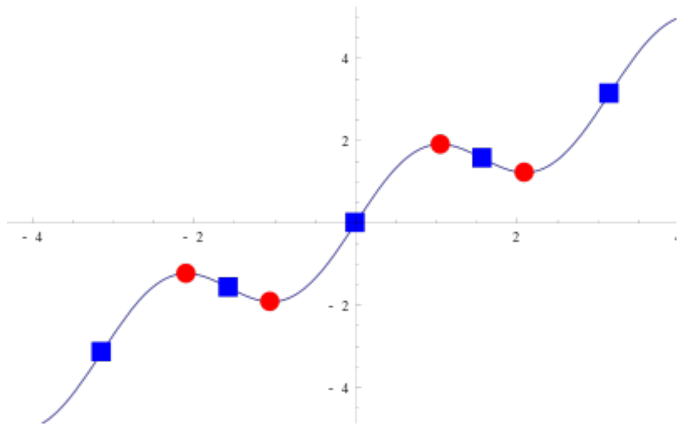

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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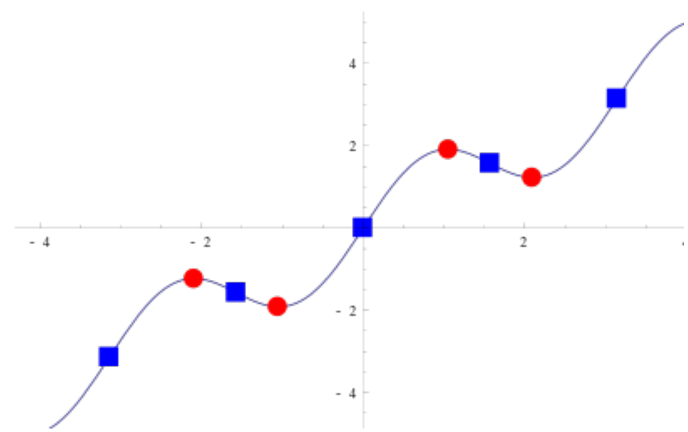

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Critical Points and Inverses

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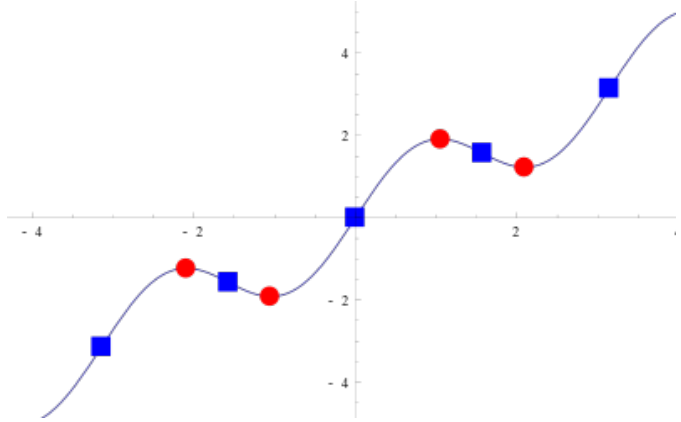
(Demo)

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that f(x) = y



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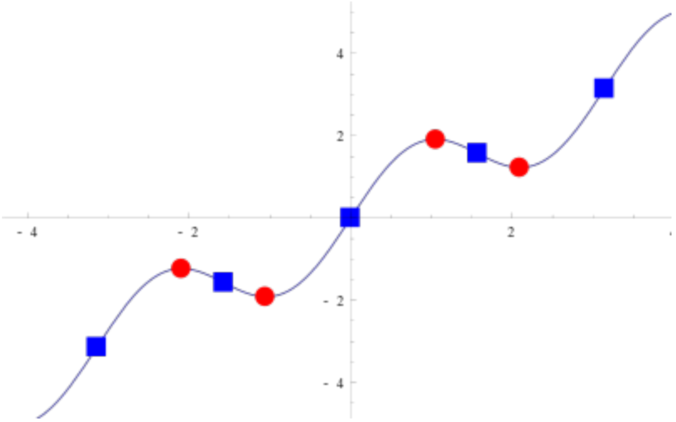

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