61A Lecture 31

Announcements

Efficient Sequence Processing

Sequence Operations

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Map, filter, and reduce express sequence manipulation using compact expressions

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Example: Sum all primes in an interval from **a** (inclusive) to **b** (exclusive)

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Space: $\Theta(1)$

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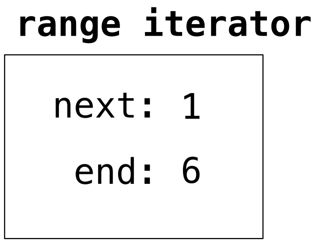

range iterator

next: 1

end: 6

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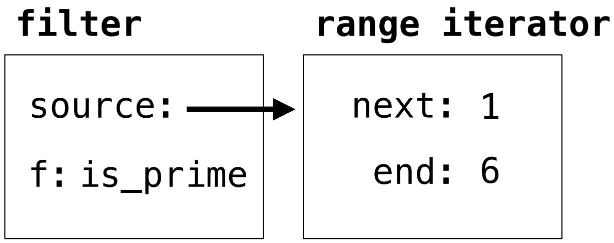

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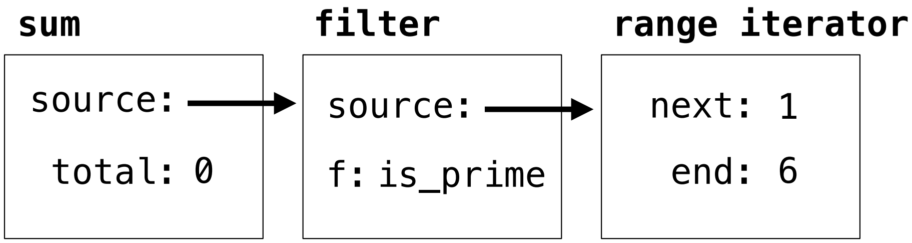

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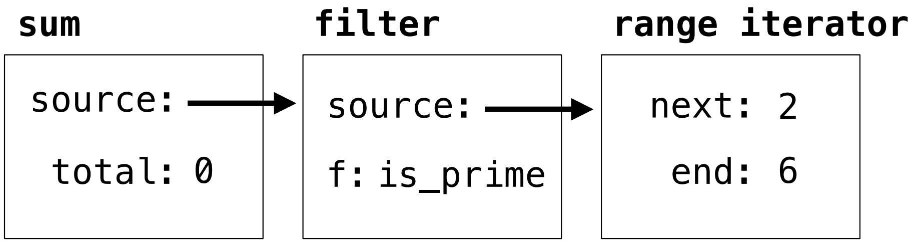

sum

source:

total: 0

filter

source:

f: is_prime

range iterator

next: 2

end: 6

Space: $\Theta(1)$

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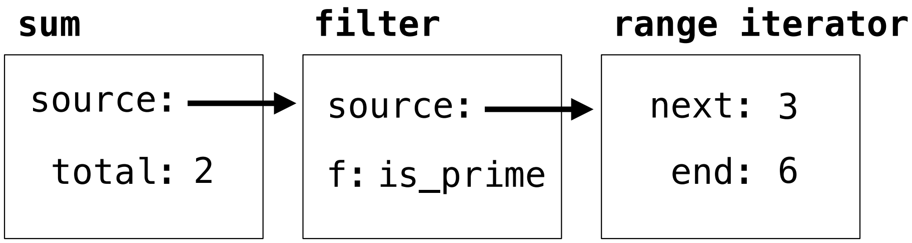

sum

source:

total: 2

filter

source:

f: is_prime

range iterator

next: 3

end: 6

Space: $\Theta(1)$

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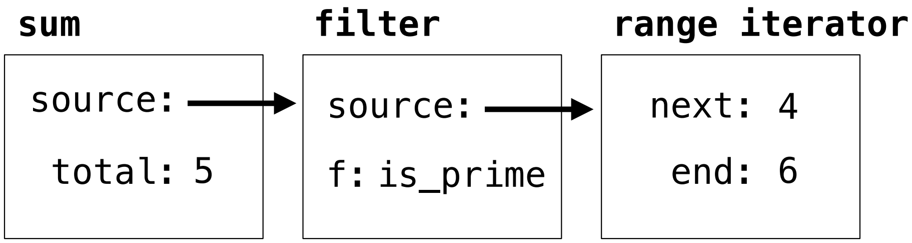
sum

source:

total: 5

filter

source:

f: is_prime

range iterator

next: 4

end: 6

Space: $\Theta(1)$

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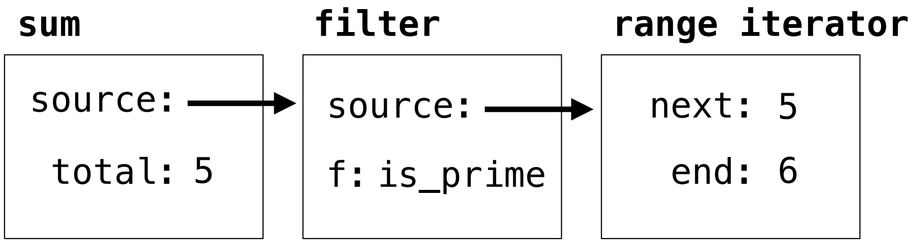

sum

source:

total: 5

filter

source:

f: is_prime

range iterator

next: 5

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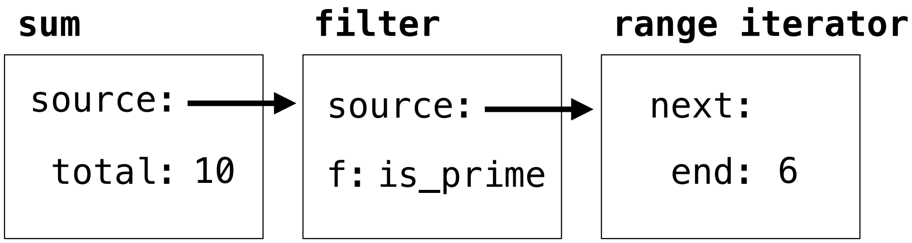

sum

source:

total: 10

filter

source:

f: is_prime

range iterator

next:

end: 6

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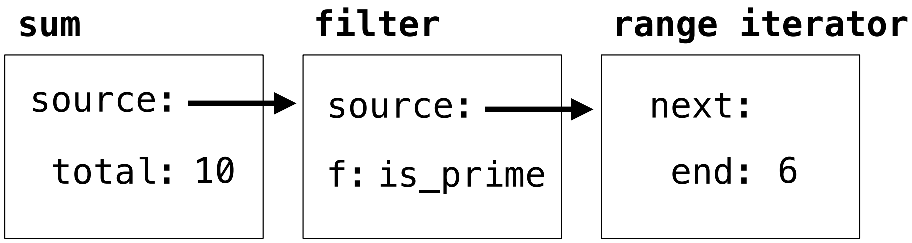
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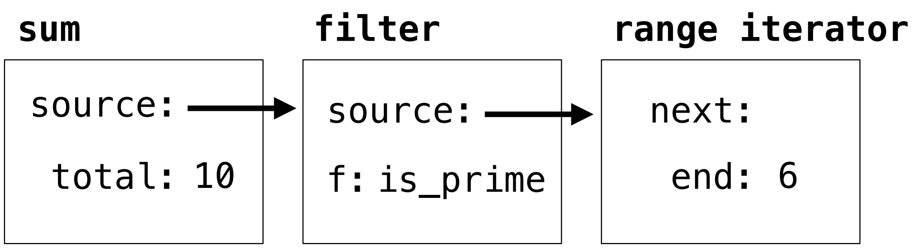

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(Demo)

Streams

Streams are Lazy Scheme Lists

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A stream is a list, but the rest of the list is computed only when needed:

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A stream is a list, but the rest of the list is computed only when needed:

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(car (cons 1 2)) -> 1                (car        (cons-stream 1 2)) -> 1

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```
(cons 1 (/ 1 0))       -> ERROR
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(Demo)

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scm> (car lots)
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scm> (car (cdr-stream lots))
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scm> (car (cdr-stream (cdr-stream lots)))
3
```

Infinite Streams

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(Demo)

Stream Processing

(Demo)

Recursively Defined Streams

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1 1 1 1 1 1 ...

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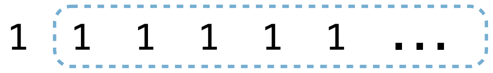

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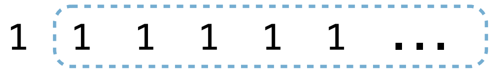

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Combine two streams by separating each into car and cdr

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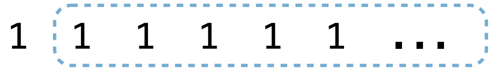

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Combine two streams by separating each into car and cdr

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(define (add-streams s t)
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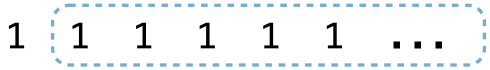
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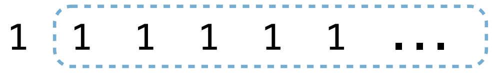
1 1 1 1 1 1 ...

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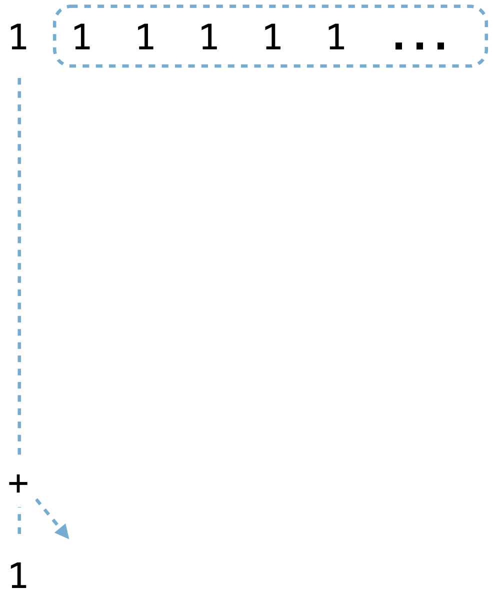

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+

1

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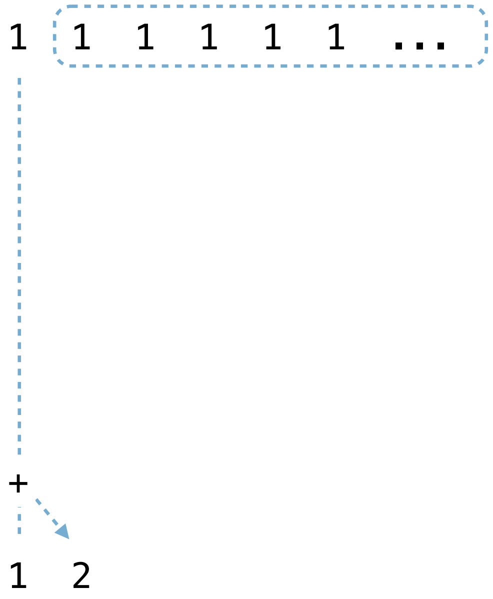

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+

1 2

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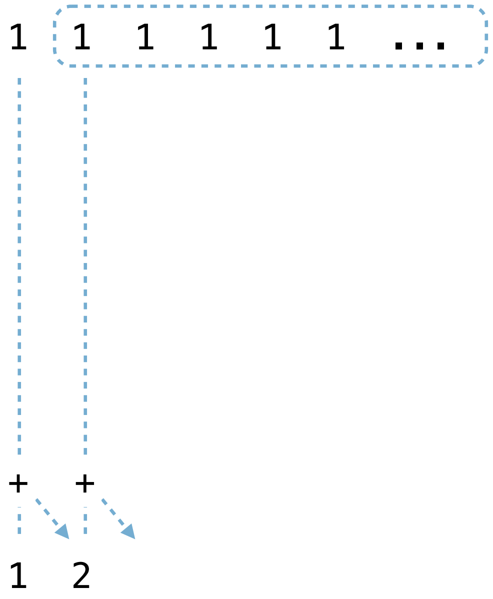
(define ones (cons-stream 1 ones))
```

Combine two streams by separating each into car and cdr

```
(define (add-streams s t)
  (cons-stream (+ (car s) (car t))
               (add-streams (cdr-stream s)
                            (cdr-stream t))))
```

```
(define ints (cons-stream 1 (add-streams ones ints)))
```



Recursively Defined Streams

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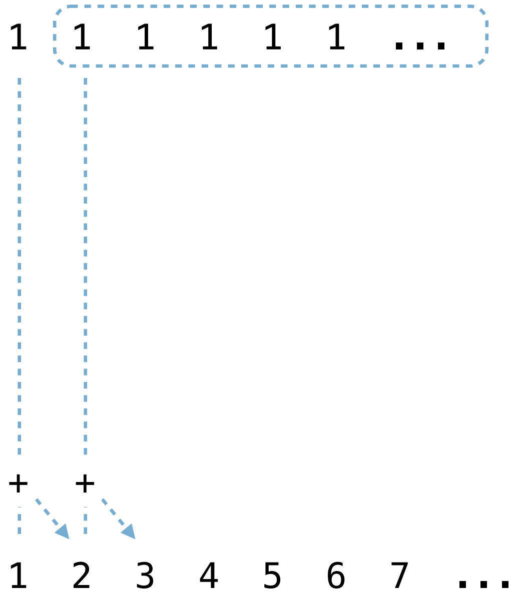

Example: Repeats

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```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))
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```

```
What's (prefix a 8)?      ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))
```

What's (prefix a 8)?   (__ __ __ __ __ __ __ __)

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))
```

```
What's (prefix a 8)?      ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?  ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?       ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?   ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

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(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
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(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?       ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?   ( __  __  __  __  __  __  __  __ )

What's (prefix (g a) 8)?   ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
                           1   2   3
What's (prefix a 8)?      ( __  __  __  __  __  __  __  __ )

What's (prefix (f a) 8)?  ( __  __  __  __  __  __  __  __ )

What's (prefix (g a) 8)?  ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
                            1    2    3    1    2    3    1    2
What's (prefix a 8)?      ( __   __   __   __   __   __   __   __ )

What's (prefix (f a) 8)?  ( __   __   __   __   __   __   __   __ )

What's (prefix (g a) 8)?  ( __   __   __   __   __   __   __   __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?      ( 1  2  3  1  2  3  1  2 )

What's (prefix (f a) 8)?  ( 1  __ __ __ __ __ __ __ )

What's (prefix (g a) 8)?  ( __ __ __ __ __ __ __ __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?      ( 1  2  3  1  2  3  1  2 )

What's (prefix (f a) 8)?  ( 1  1  __ __ __ __ __ __ )

What's (prefix (g a) 8)?  ( __ __ __ __ __ __ __ __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
                             1    2    3    1    2    3    1    2
What's (prefix a 8)?       ( __   __   __   __   __   __   __   __ )

                             1    1    2
What's (prefix (f a) 8)?   ( __   __   __   __   __   __   __   __ )

What's (prefix (g a) 8)?   ( __   __   __   __   __   __   __   __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
                           1    2    3    1    2    3    1    2
What's (prefix a 8)?     ( __   __   __   __   __   __   __   __ )

                           1    1    2    2
What's (prefix (f a) 8)? ( __   __   __   __   __   __   __   __ )

What's (prefix (g a) 8)? ( __   __   __   __   __   __   __   __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?       ( 1   2   3   1   2   3   1   2  )

What's (prefix (f a) 8)?   ( 1   1   2   2   3   3   1   1  )

What's (prefix (g a) 8)?   ( __  __  __  __  __  __  __  __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

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```

```
What's (prefix a 8)?       ( 1  2  3  1  2  3  1  2 )

What's (prefix (f a) 8)?   ( 1  1  2  2  3  3  1  1 )

What's (prefix (g a) 8)?   ( 1  __ __ __ __ __ __ __ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

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```
What's (prefix a 8)?      ( 1  2  3  1  2  3  1  2 )

What's (prefix (f a) 8)?  ( 1  1  2  2  3  3  1  1 )

What's (prefix (g a) 8)?  ( 1  2  2  __ __ __ __ __ )
```

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(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

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                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?      ( _1_ _2_ _3_ _1_ _2_ _3_ _1_ _2_ )

What's (prefix (f a) 8)?  ( _1_ _1_ _2_ _2_ _3_ _3_ _1_ _1_ )

What's (prefix (g a) 8)?  ( _1_ _2_ _2_ _3_ _3_ ___ ___ ___ )
```

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

(define (f s) (cons-stream (car s)
                           (cons-stream (car s)
                                        (f (cdr-stream s)))))

(define (g s) (cons-stream (car s)
                           (f (g (cdr-stream s)))))
```

What's (prefix a 8)? (1 2 3 1 2 3 1 2)

What's (prefix (f a) 8)? (1 1 2 2 3 3 1 1)

What's (prefix (g a) 8)? (1 2 2 3 3 3 3 __)

Example: Repeats

```
(define a (cons-stream 1 (cons-stream 2 (cons-stream 3 a))))

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                           (f (g (cdr-stream s)))))
```

```
What's (prefix a 8)?       ( 1  2  3  1  2  3  1  2 )

What's (prefix (f a) 8)?   ( 1  1  2  2  3  3  1  1 )

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```

Higher-Order Stream Functions

Higher-Order Functions on Streams

Implementations are identical,
but change cons to cons-stream
and change cdr to cdr-stream

Higher-Order Functions on Streams

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```scheme
(define (map f s)
  (if (null? s)
      nil
      (cons (f (car s))
            (map f
                 (cdr s)))))

(define (filter f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons (car s)
                (filter f (cdr s)))
          (filter f (cdr s)))))

(define (reduce f s start)
  (if (null? s)
      start
      (reduce f
              (cdr s)
              (f start (car s)))))
```

Higher-Order Functions on Streams

```
Implementations are identical,
but change cons to cons-stream
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```

```
(define (map        f s)
  (if (null? s)
      nil
      (cons        (f (car s))
            (map        f
                 (cdr        s)))))

(define (filter        f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons        (car s)
                (filter        f (cdr        s)))
          (filter        f (cdr        s)))))

(define (reduce        f s start)
  (if (null? s)
      start
      (reduce        f
              (cdr        s)
              (f start (car s)))))
```

Higher-Order Functions on Streams

Implementations are identical, but change cons to cons–stream and change cdr to cdr–stream

```
(define (map-stream f s)
  (if (null? s)
      nil
      (cons-stream (f (car s))
            (map-stream f
                 (cdr-stream s)))))

(define (filter-stream f s)
  (if (null? s)
      nil
      (if (f (car s))
          (cons-stream (car s)
                (filter-stream f (cdr-stream s)))
          (filter-stream f (cdr-stream s)))))

(define (reduce-stream f s start)
  (if (null? s)
      start
      (reduce-stream f
              (cdr-stream s)
              (f start (car s)))))
```

A Stream of Primes

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```
For any prime k, any larger prime must not be divisible by k.
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The stream of integers not divisible by any k <= n is:

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▲

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(Demo)

Promises

Implementing Streams with Delay and Force

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A promise is an expression, along with an environment in which to evaluate it

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```
scm> (define promise (let ((x 2)) (delay     (+ x 1)) ))
```

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```
scm> (define promise (let ((x 2)) (delay     (+ x 1)) ))


scm> (force promise)
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```

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```
scm> (define promise (let ((x 2)) (delay     (+ x 1)) ))

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```
(define-macro (delay expr)    `(lambda () ,expr))
(define       (force promise) (promise))
```

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A stream is a list, but the rest of the list is computed only when **forced:**

```
scm> (define ones (cons-stream 1 ones))
```

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```

A stream is a list, but the rest of the list is computed only when **forced:**

```
scm> (define ones (cons-stream 1 ones))
(1 . #[promise (not forced)])
```

Implementing Streams with Delay and Force

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```

```
(define-macro (cons-stream a b) `(cons ,a (delay ,b)))
(define       (cdr-stream s)     (force (cdr s)))
```

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```
(define-macro (delay expr)    `(lambda () ,expr))
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A stream is a list, but the rest of the list is computed only when **forced:**

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scm> (define ones (cons-stream 1 ones))
(1 . #[promise (not forced)])
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```