Ordered Sets

Announcements

Sets

Sets

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One more built-in Python container type

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```
>>> s = {'one', 'two', 'three', 'four', 'four'}
```

Sets

One more built-in Python container type

- Set literals are enclosed in braces
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```
{'three', 'one', 'four', 'two'}
```

Sets

One more built-in Python container type

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```
True
```

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4

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```
{'three', 'five', 'one', 'four', 'two'}
```

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```
{'three', 'four'}
```

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Implementing Sets

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What we should be able to do with a set:

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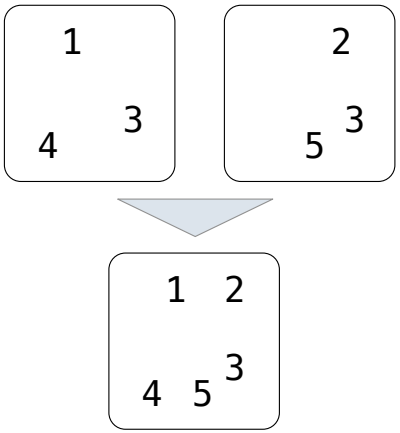
Union

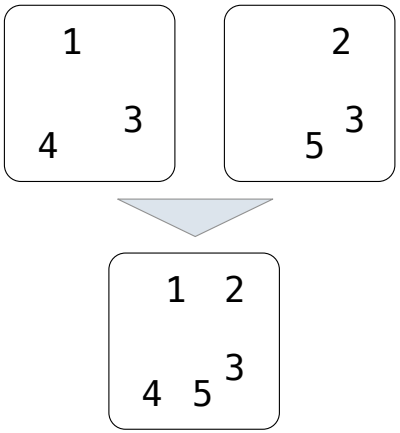


Implementing Sets

What we should be able to do with a set:

- **Membership testing:** Is a value an element of a set?
- **Union:** Return a set with all elements in set1 or set2
- **Intersection:** Return a set with any elements in set1 and set2

Union



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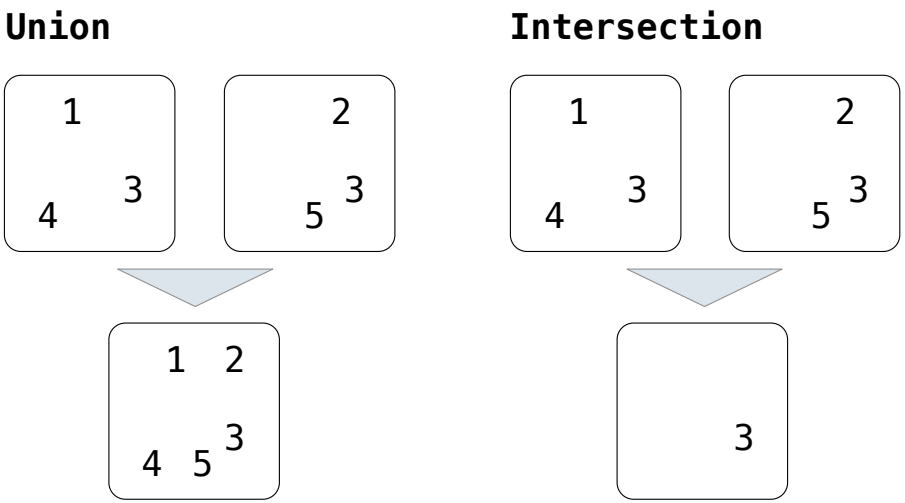

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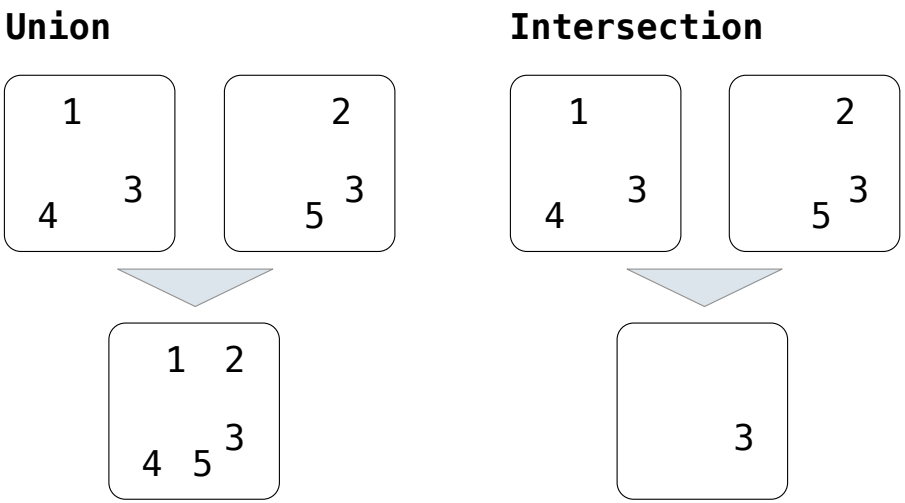

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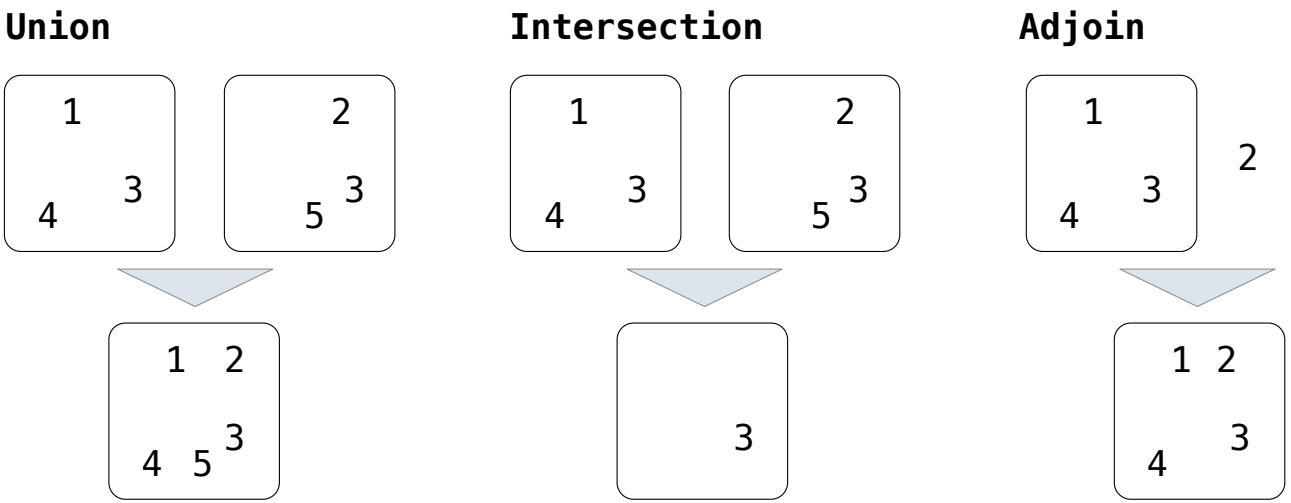

Sets as Linked Lists

Sets as Unordered Sequences

Proposal 1: A set is represented by a linked list that contains no duplicate items.

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(Demo)

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```

(Demo)

Time depends on whether & where v appears in s.

$\Theta(n)$

In the worst case: v does not appear in s

or

In the average case: appears in a uniformly distributed random location

Sets as Unordered Sequences

Sets as Unordered Sequences

```
def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
```

Sets as Unordered Sequences

Time order of worst-case growth

```
def adjoin(s, v):
    if contains(s, v):
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Sets as Unordered Sequences

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Time order of worst-case growth

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The size of the set

Sets as Unordered Sequences

```
def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
```

```
def intersect(s, t):
    if s is Link.empty:
        return Link.empty
    rest = ____________________________
    if contains(t, s.first):
        return ____________________________
    else:
        return rest
```

Time order of worst-case growth

$\Theta(n)$

The size of the set

Sets as Unordered Sequences

```
def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
```

Time order of worst-case growth

$\Theta(n)$

The size of the set

```
def intersect(s, t):
    if s is Link.empty:
        return Link.empty
    rest = ___intersect(s.rest, t)___
    if contains(t, s.first):
        return _____________________
    else:
        return rest
```

Sets as Unordered Sequences

```
def adjoin(s, v):
    if contains(s, v):
        return s
    else:
        return Link(v, s)
```

```
def intersect(s, t):
    if s is Link.empty:
        return Link.empty
    rest = intersect(s.rest, t)
    if contains(t, s.first):
        return Link(s.first, rest)
    else:
        return rest
```

Time order of worst-case growth

$\Theta(n)$

The size of the set

Sets as Unordered Sequences

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def adjoin(s, v):
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Time order of worst-case growth

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```
def intersect(s, t):
    if s is Link.empty:
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    rest = intersect(s.rest, t)
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```

$\Theta(n^2)$

Sets as Unordered Sequences

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def adjoin(s, v):
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    else:
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```

```
def intersect(s, t):
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    else:
        return rest
```

Time order of worst-case growth

$\Theta(n)$

The size of the set

$\Theta(n^2)$

If sets are the same size

Sets as Ordered Linked Lists

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Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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Parts of the program that...	Assume that sets are...	Using...

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Use sets to contain values		

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	

Sets as Ordered Sequences

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	`empty`, `contains`, `adjoin`, `intersect`, `union`

Sets as Ordered Sequences

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	`empty`, `contains`, `adjoin`, `intersect`, `union`
Implement set operations		

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Parts of the program that...	Assume that sets are...	Using...
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Implement set operations	Ordered linked lists	

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Parts of the program that...	Assume that sets are...	Using...
Use sets to contain values	Unordered collections	`empty`, `contains`, `adjoin`, `intersect`, `union`
Implement set operations	Ordered linked lists	`first`, `rest`, `<`, `>`, `==`

Different parts of a program may make different assumptions about data

Searching an Ordered List

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```
>>> s = Link(1, Link(3, Link(5)))
```

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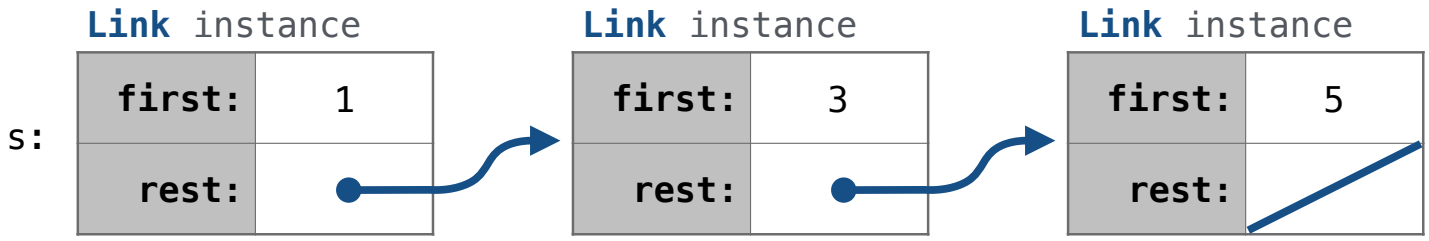

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```
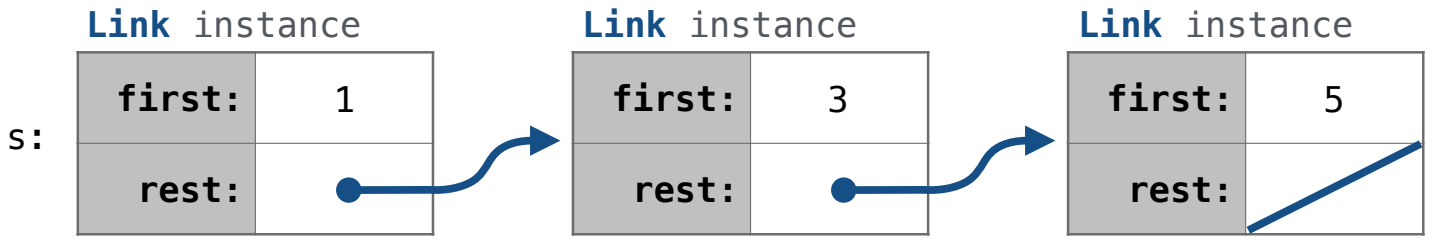

Operation	Time order of growth



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```
>>> s = Link(1, Link(3, Link(5)))
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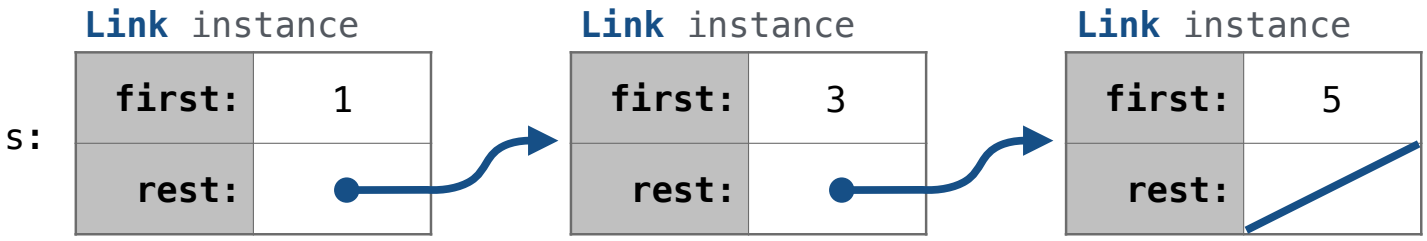
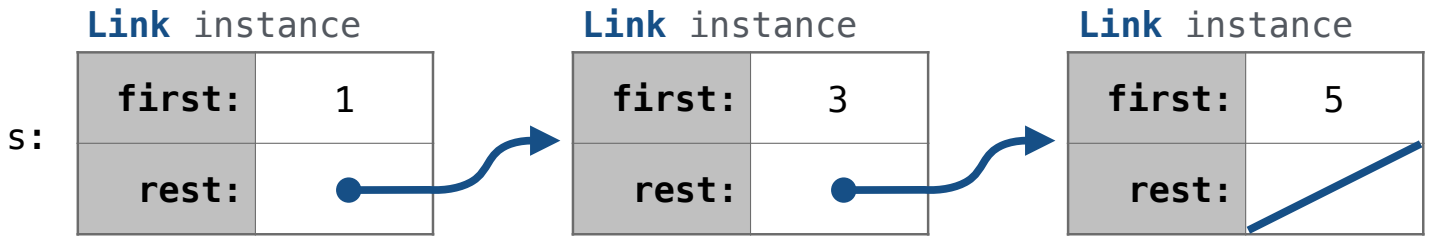
Operation	Time order of growth
contains	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
```

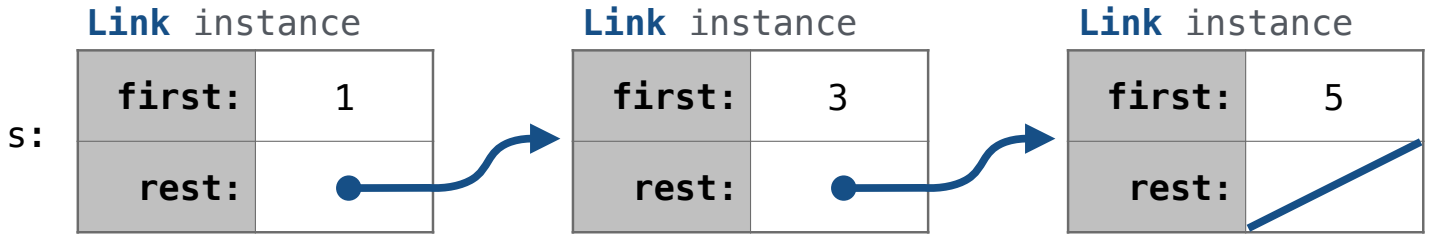
Operation	Time order of growth
contains	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
```

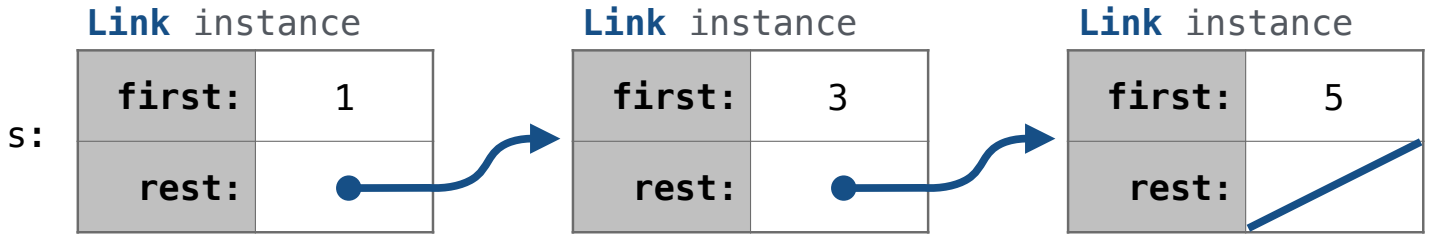
Operation	Time order of growth
contains	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
>>> contains(s, 2)
```

Operation	Time order of growth
contains	



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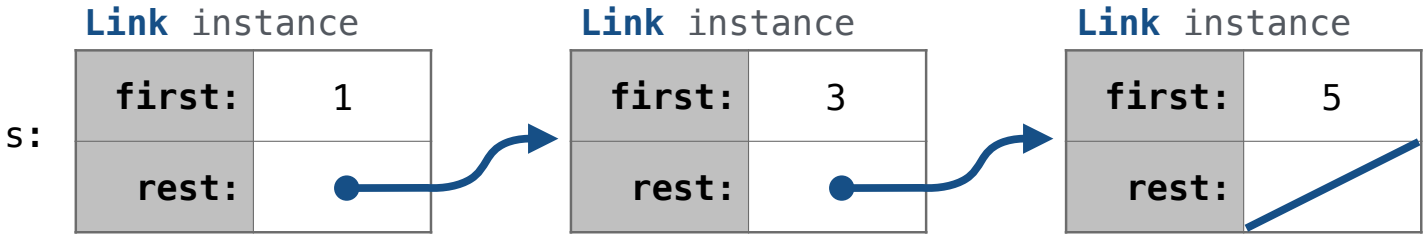

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>>> s = Link(1, Link(3, Link(5)))
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Operation	Time order of growth
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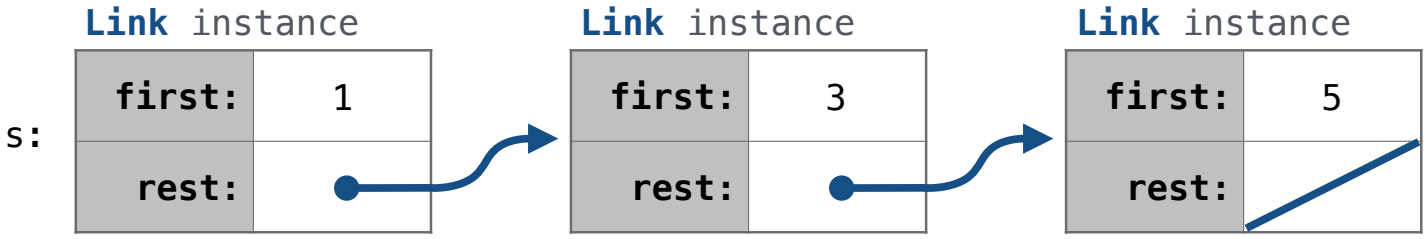

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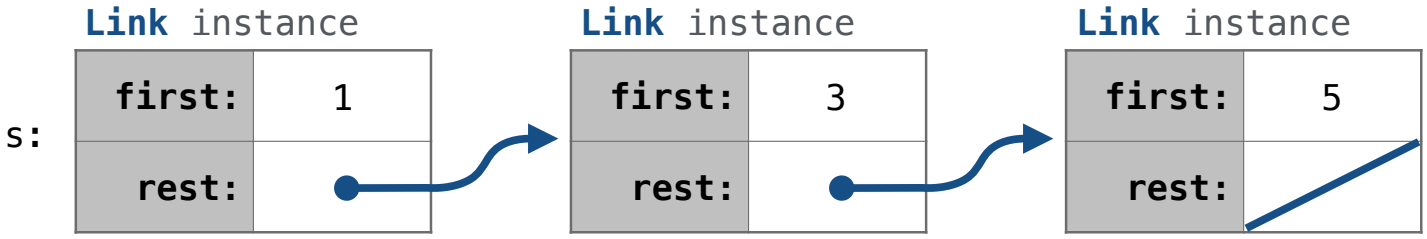

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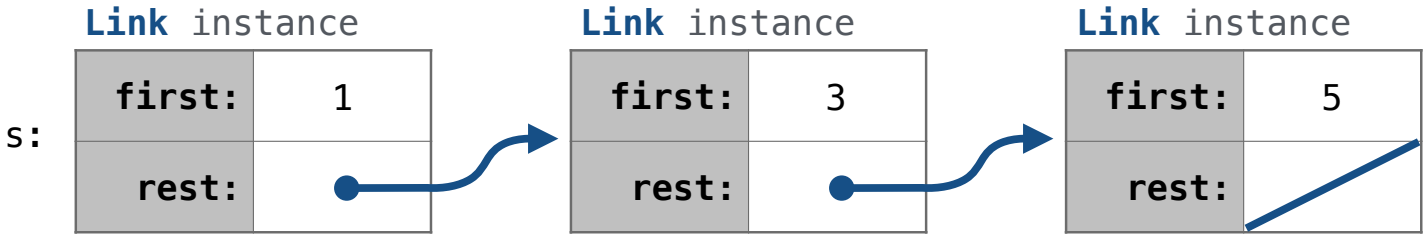

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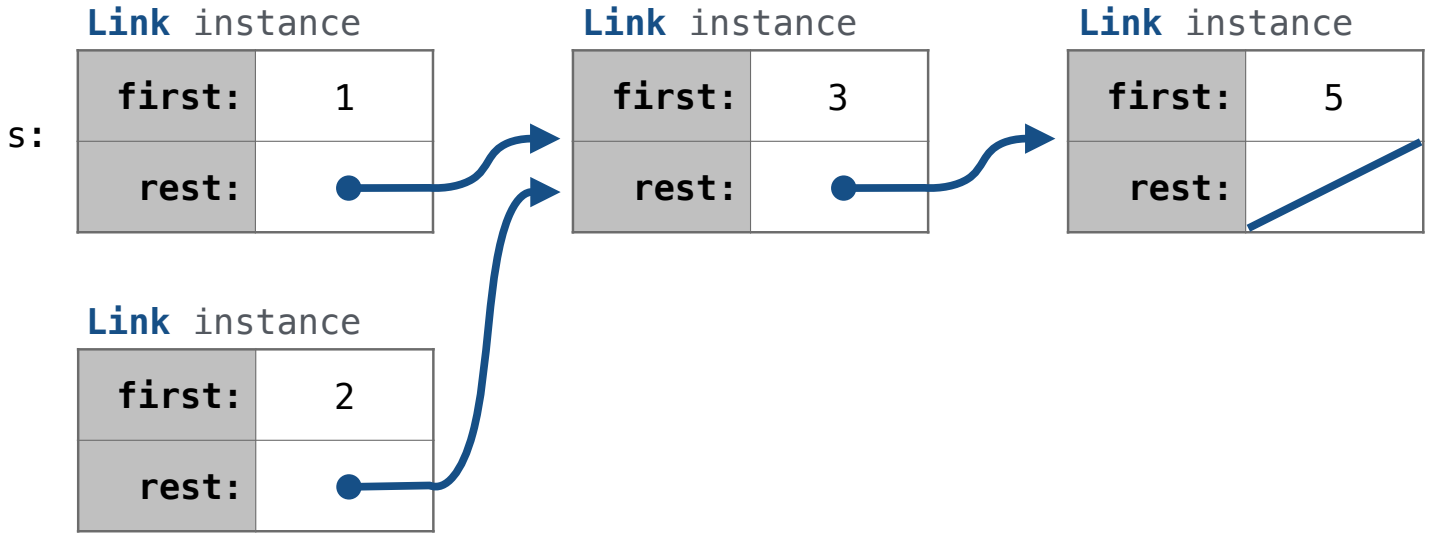

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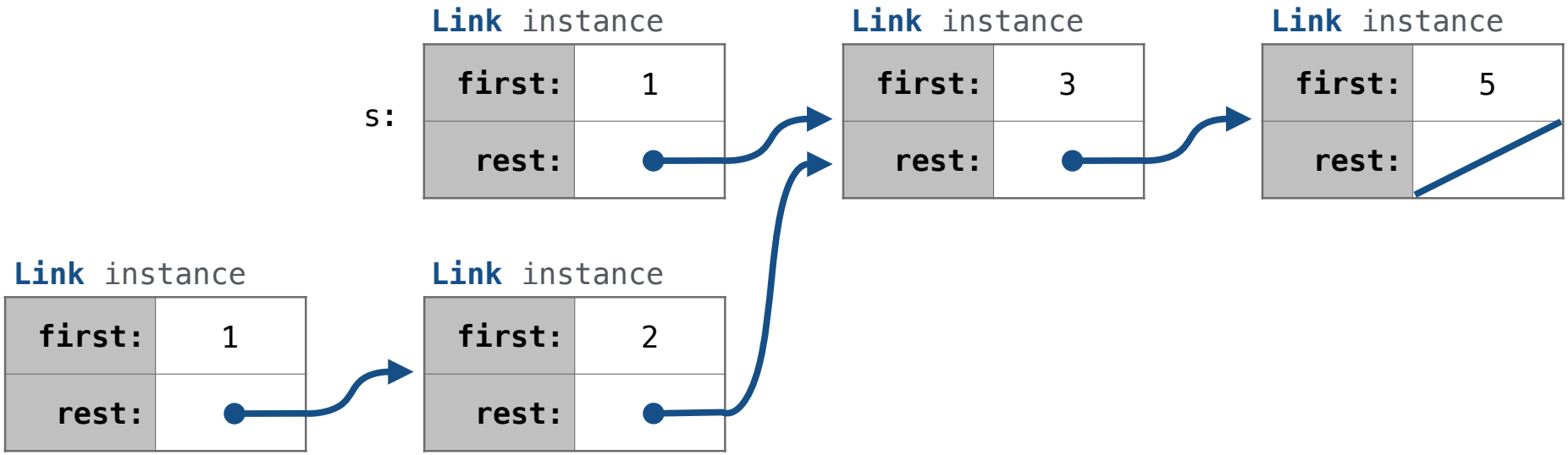

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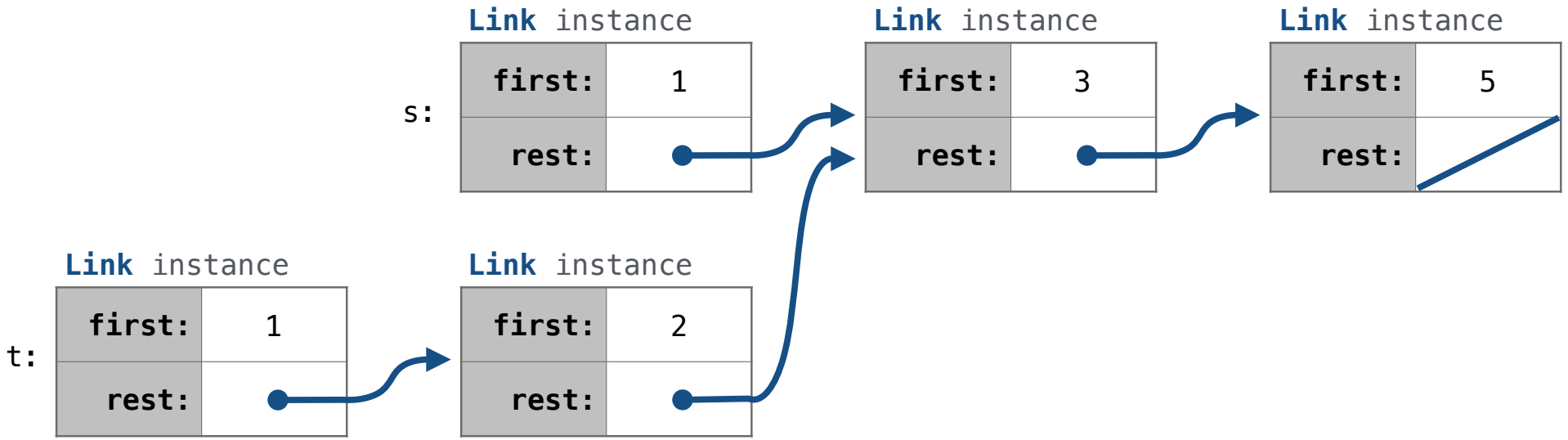

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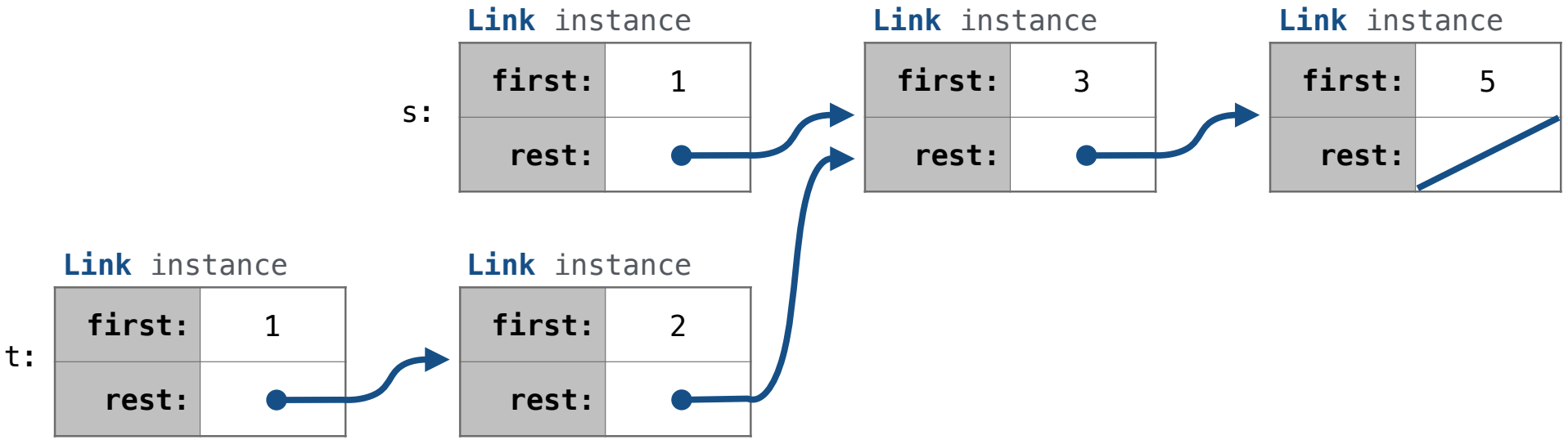

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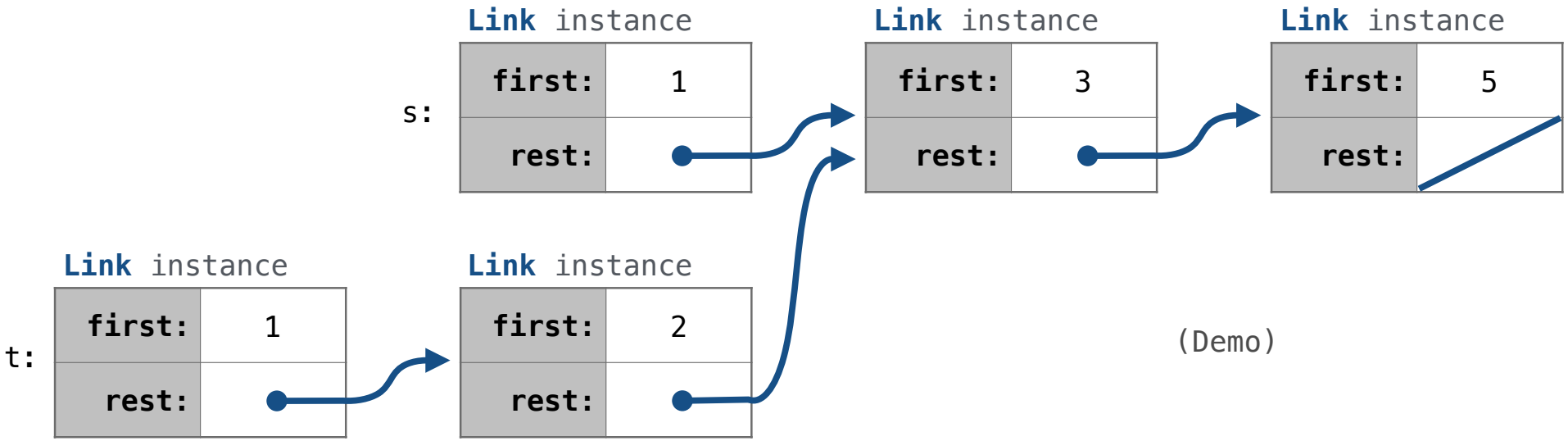

(Demo)

Set Operations

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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def intersect(s, t):
    if empty(s) or empty(t):
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    else:
        e1, e2 = s.first, t.first
```

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(s, t):
    if empty(s) or empty(t):
        return Link.empty
    else:
        e1, e2 = s.first, t.first
        if e1 == e2:
            return Link(e1, intersect(s.rest, t.rest))
```

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(s, t):
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        return Link.empty
    else:
        e1, e2 = s.first, t.first
        if e1 == e2:
            return Link(e1, intersect(s.rest, t.rest))
        elif e1 < e2:
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Intersecting Ordered Linked Lists

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Order of growth?

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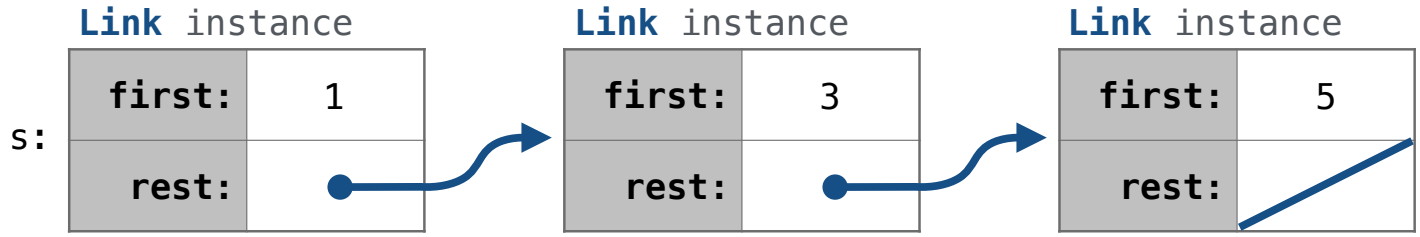
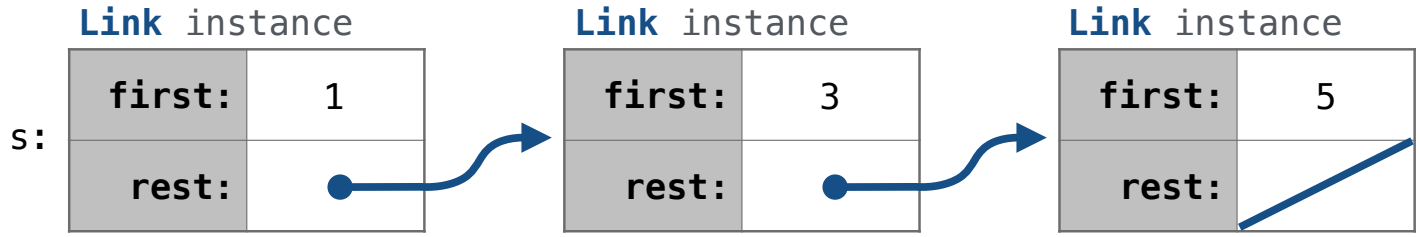
Order of growth? If s and t are sets of size n, then $\Theta(n)$ (Demo)

Set Mutation

Adding to an Ordered List

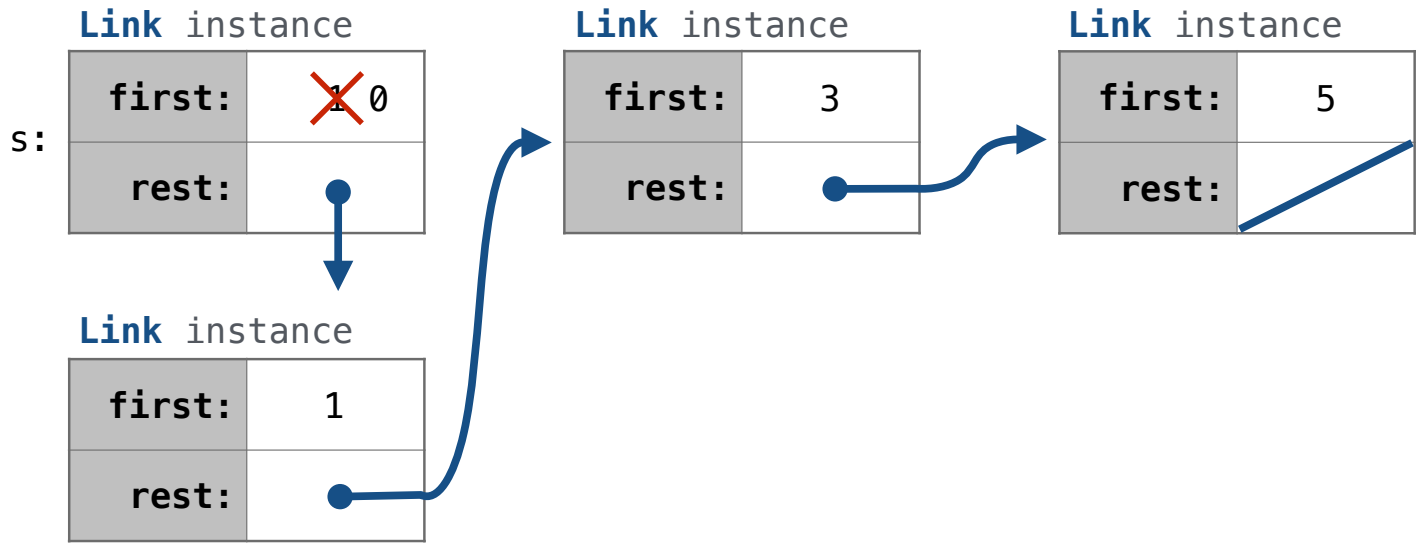


Adding to an Ordered List

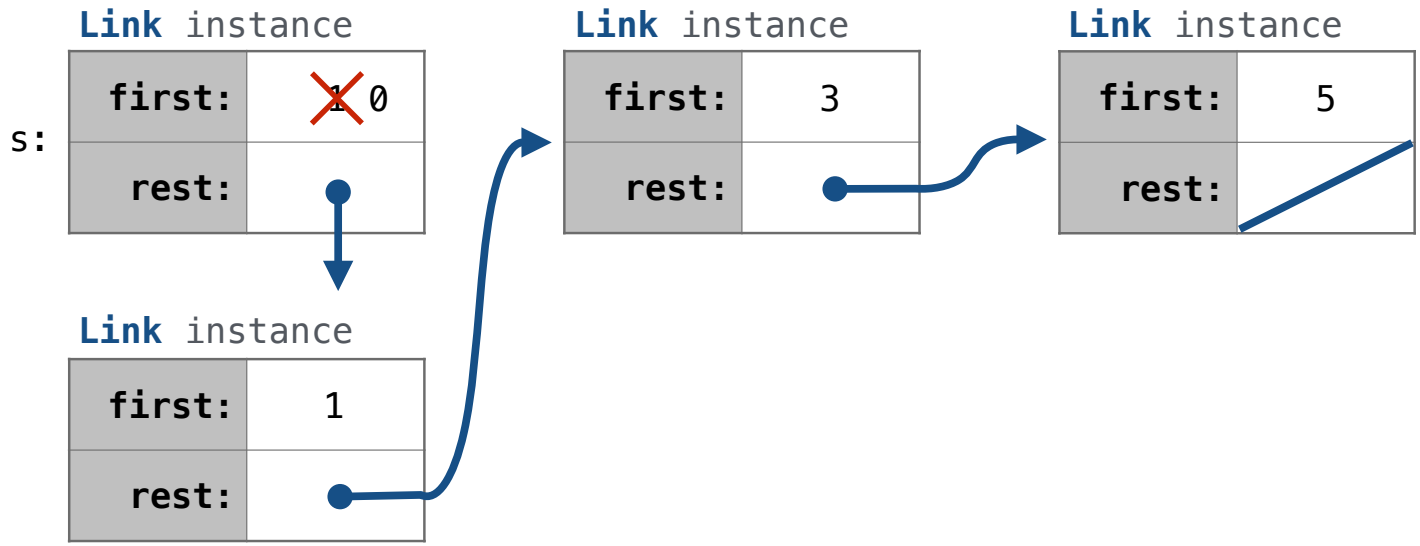



```
add(s, 0)   Try to return the same object as input
```

Adding to an Ordered List

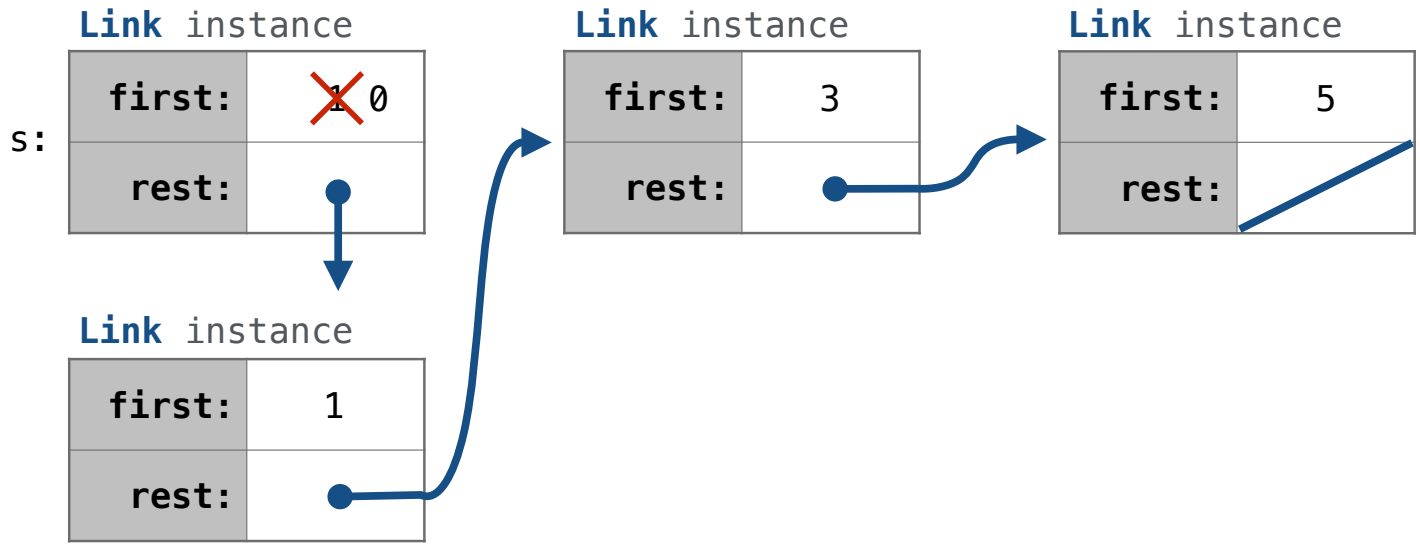


Adding to an Ordered List



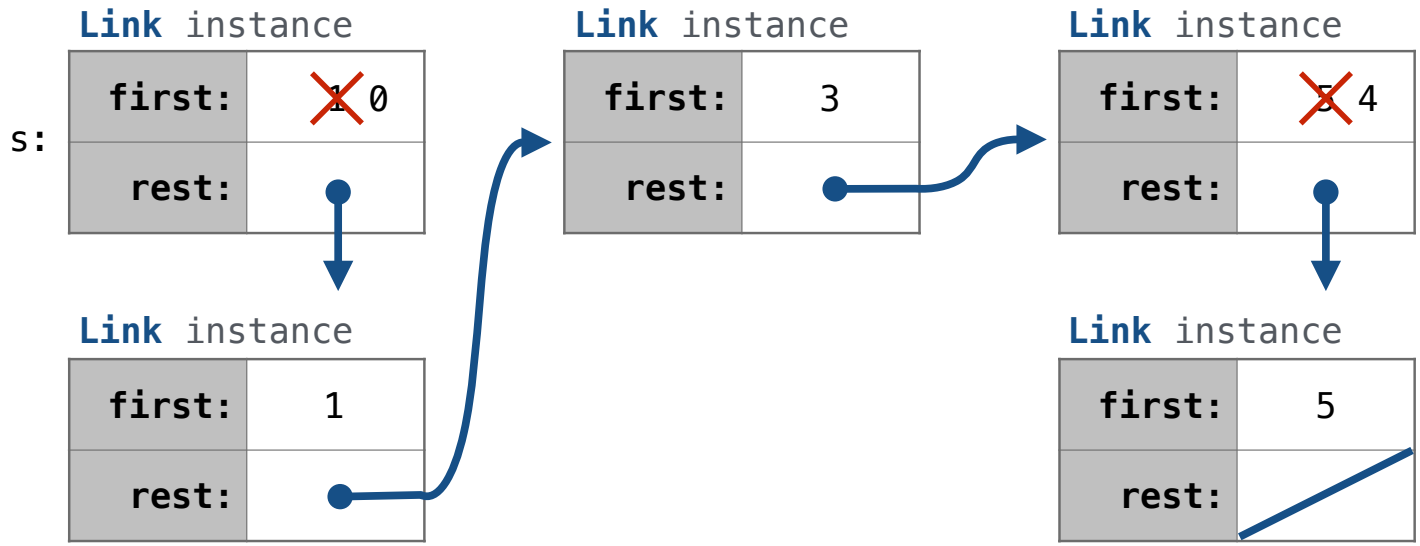

```
add(s, 3)
```

Adding to an Ordered List

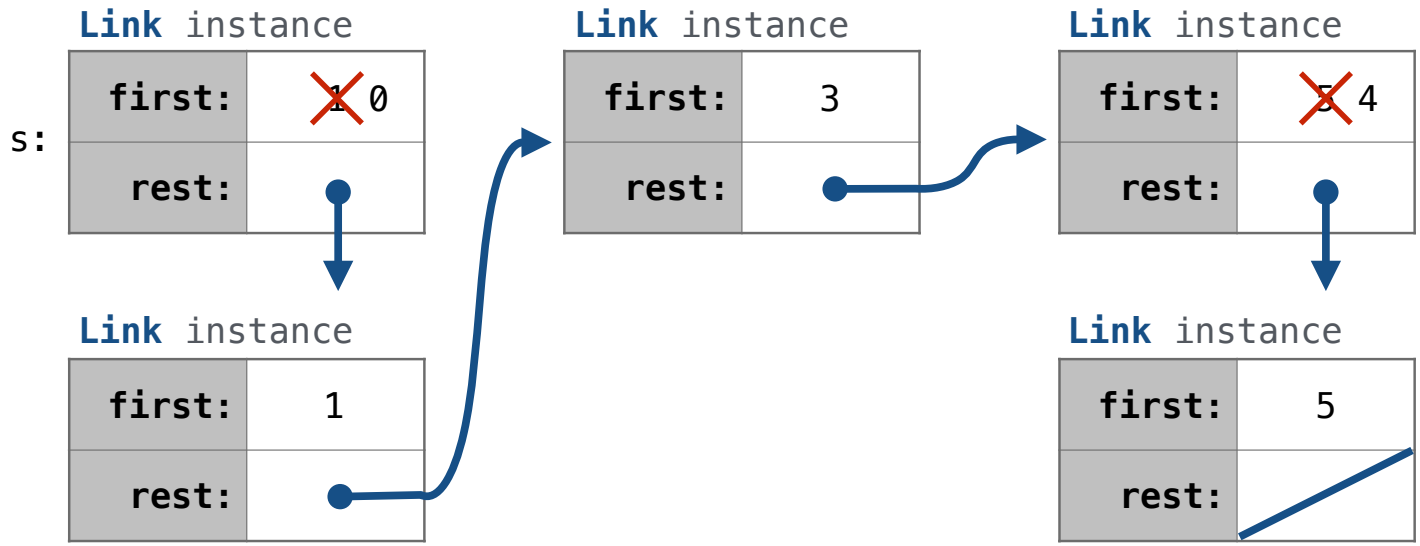



```
add(s, 3)
add(s, 4)
```

Adding to an Ordered List

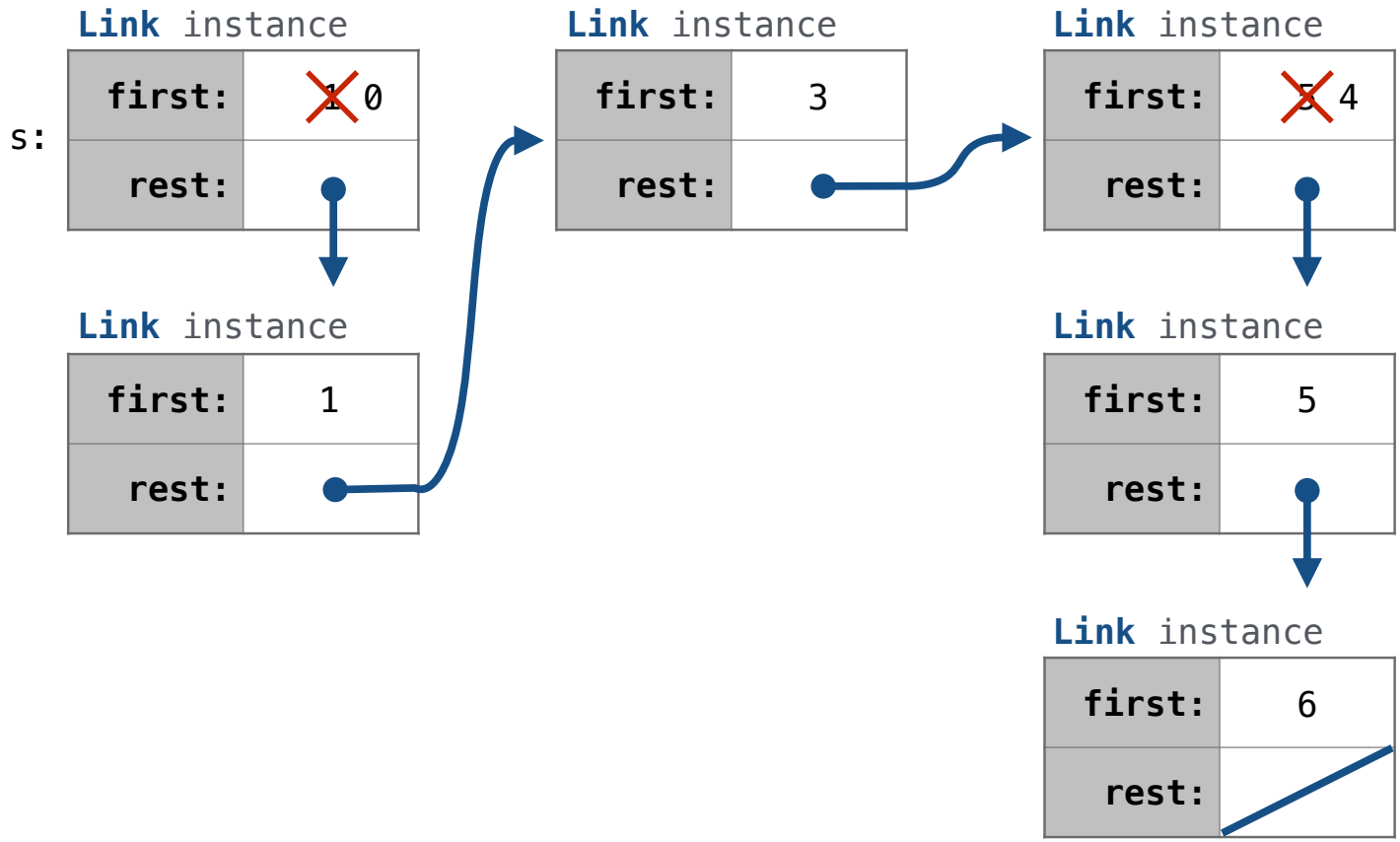


Adding to an Ordered List

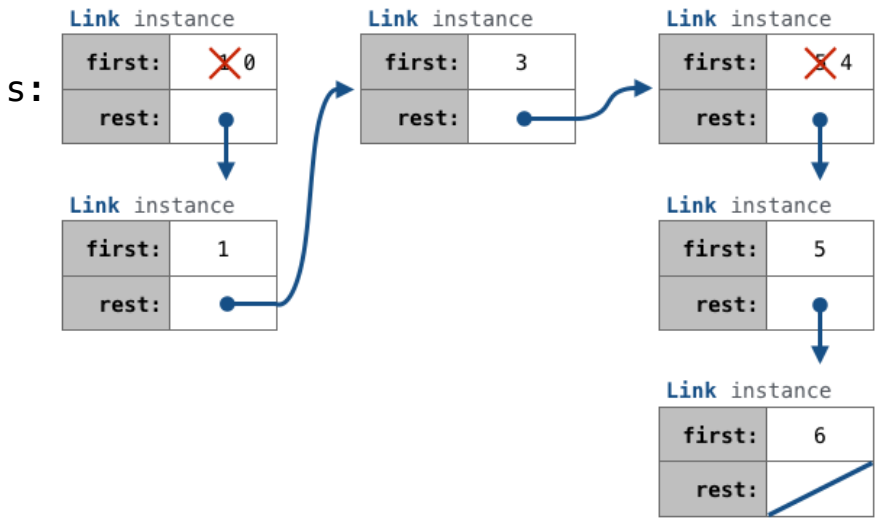



```
add(s, 6)
```

Adding to an Ordered List

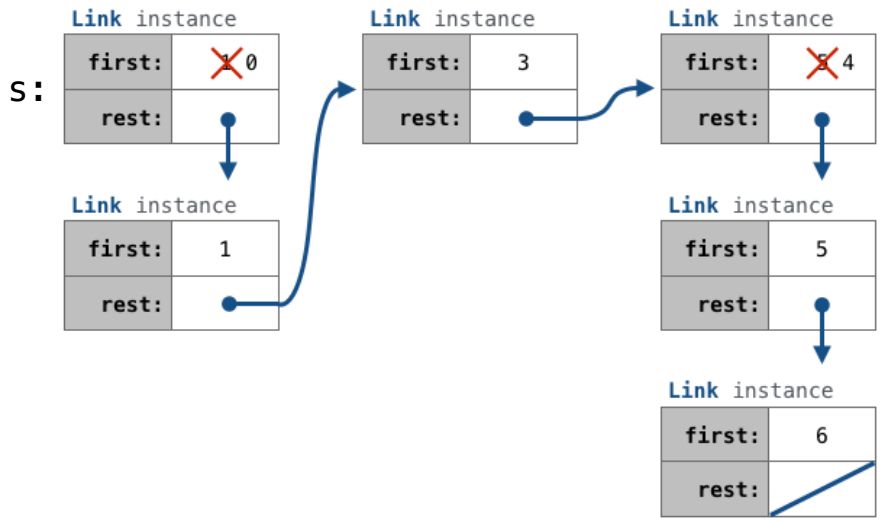


Adding to a Set Represented as an Ordered List



Adding to a Set Represented as an Ordered List

```
def add(s, v):
```



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""
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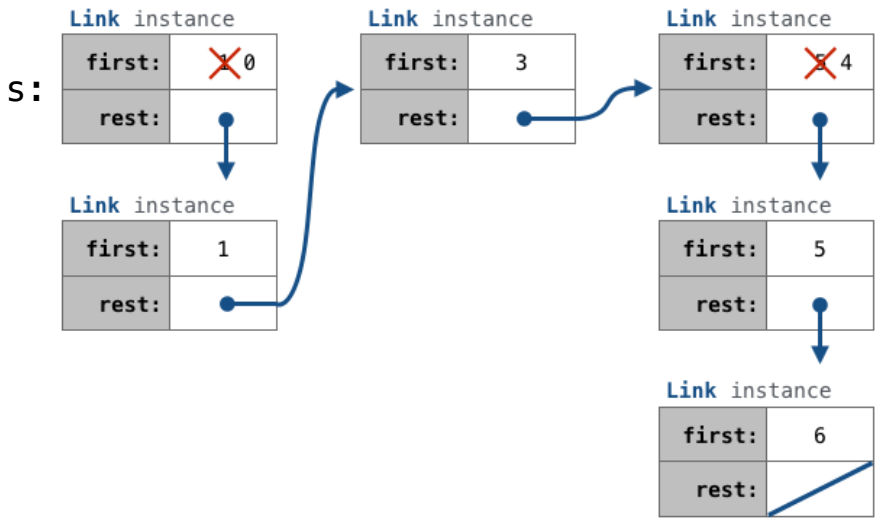

Adding to a Set Represented as an Ordered List

```
def add(s, v):
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    >>> s = Link(1, Link(3, Link(5)))
```
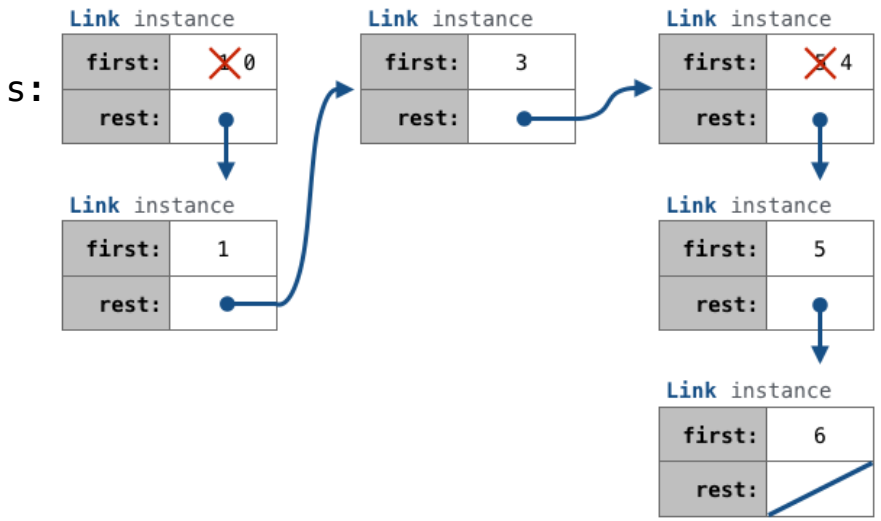
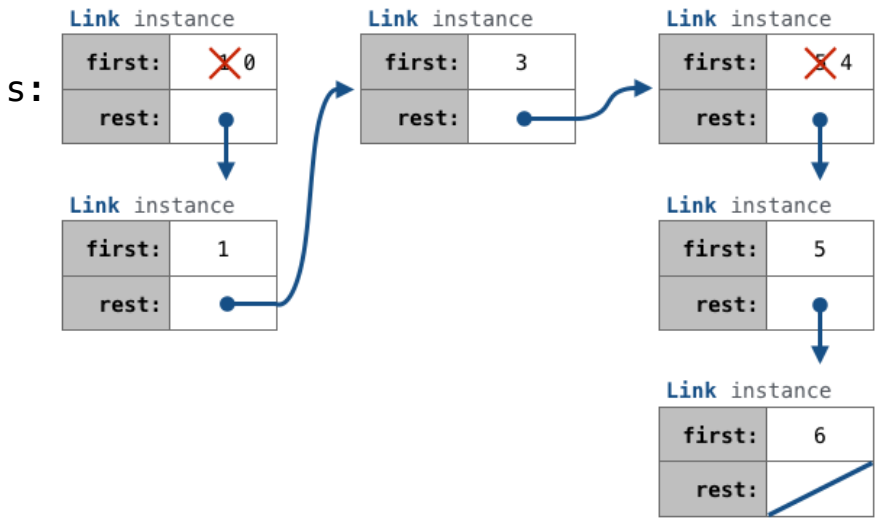



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
```



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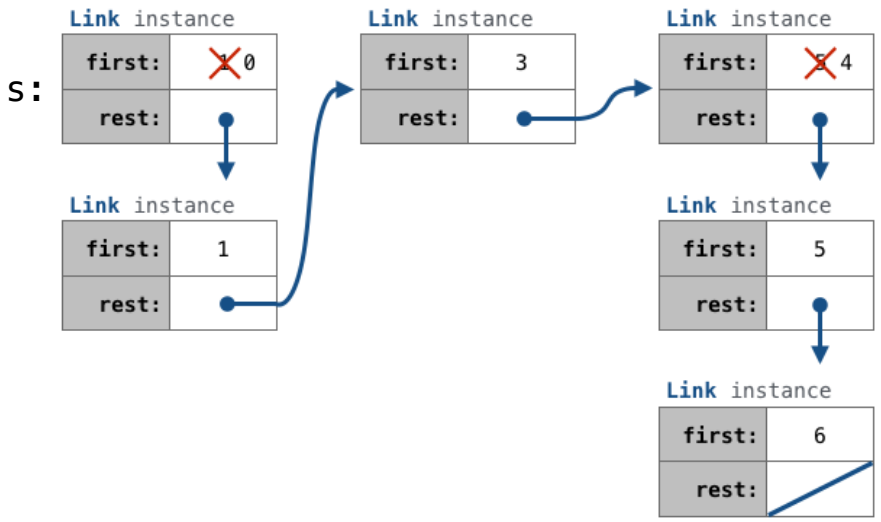

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    >>> add(s, 4)
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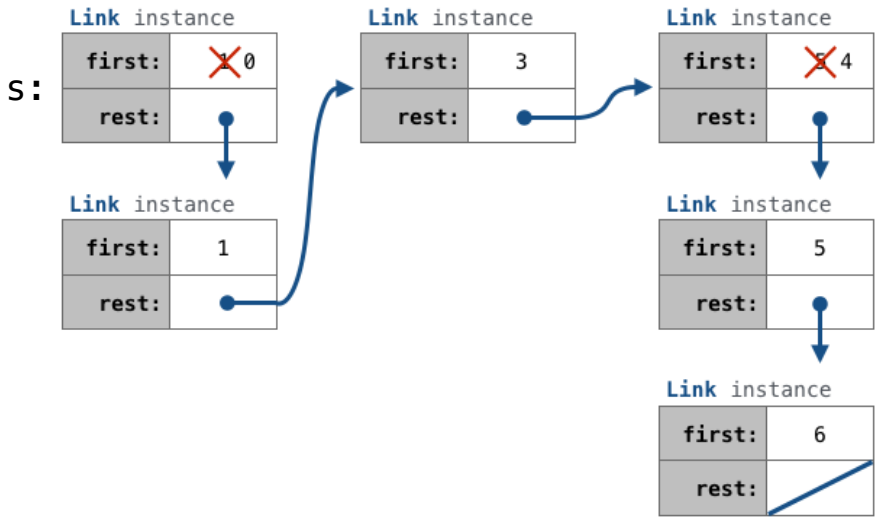

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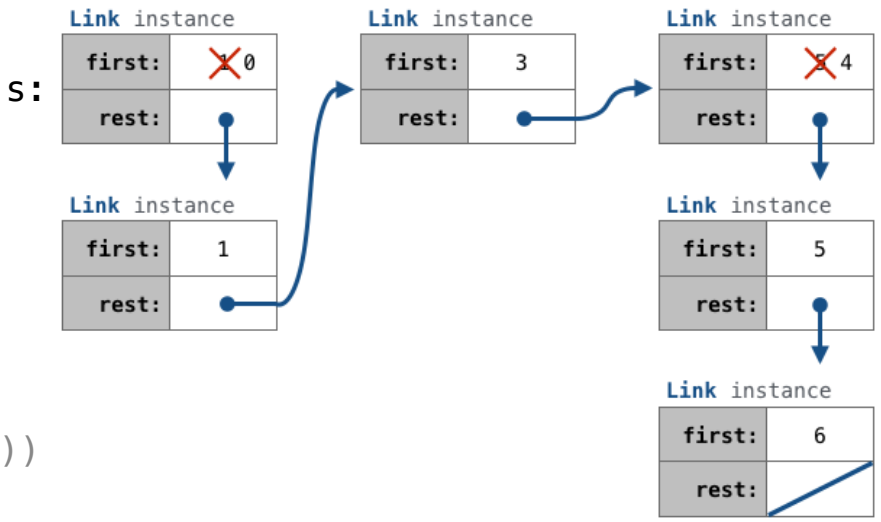

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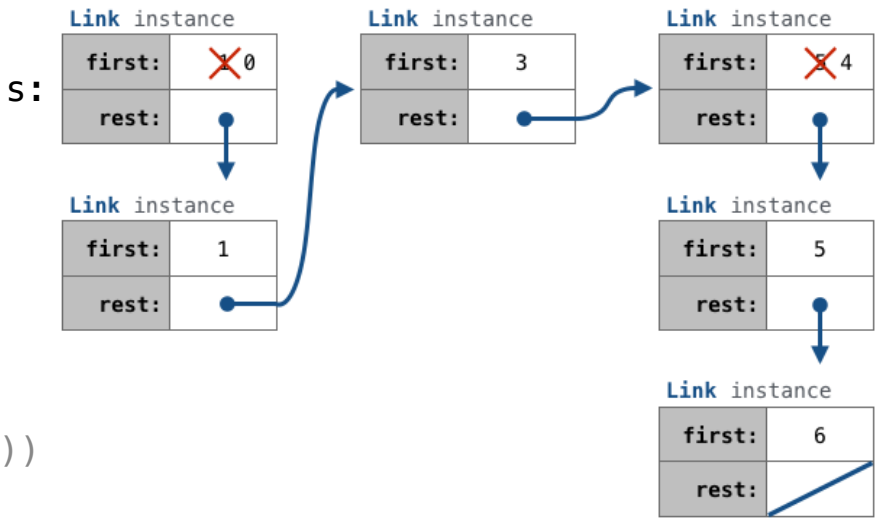
    """
    if empty(s): return Link(v)
```



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    """

    if s.first > v:
        s.first, s.rest = _____________________________ , _____________________________
```

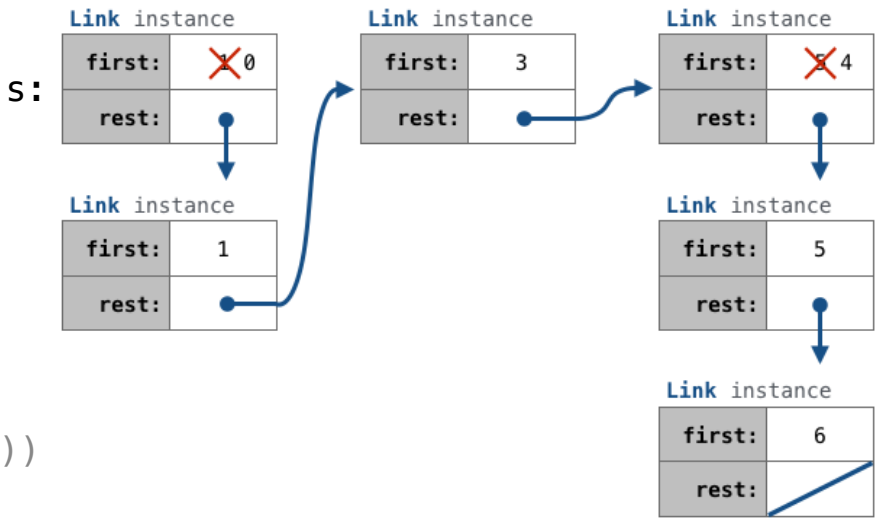

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    if s.first > v:
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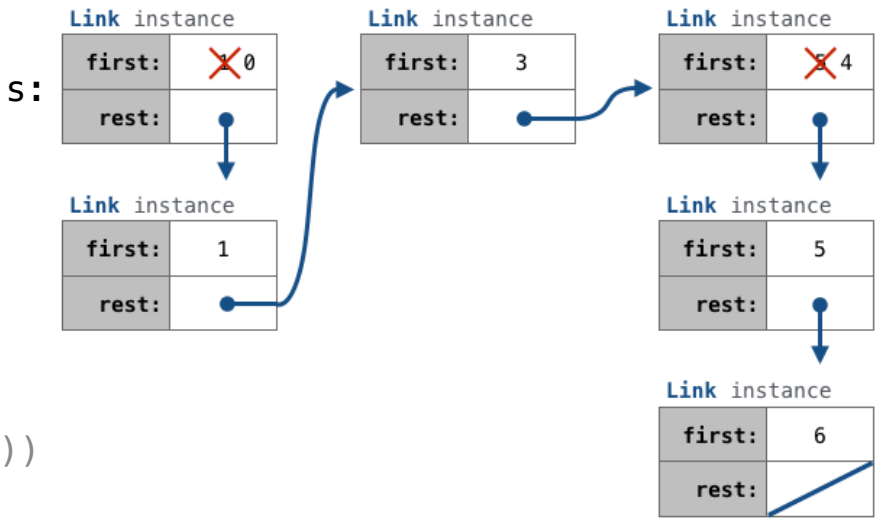

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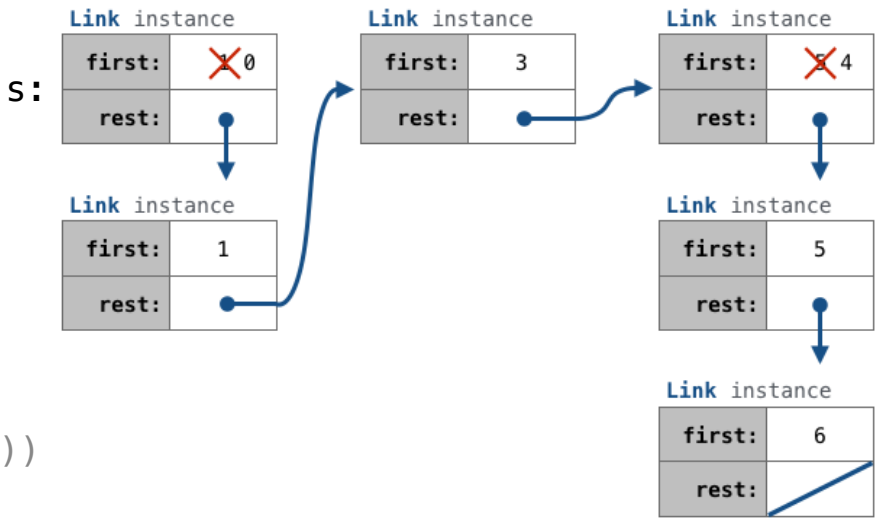

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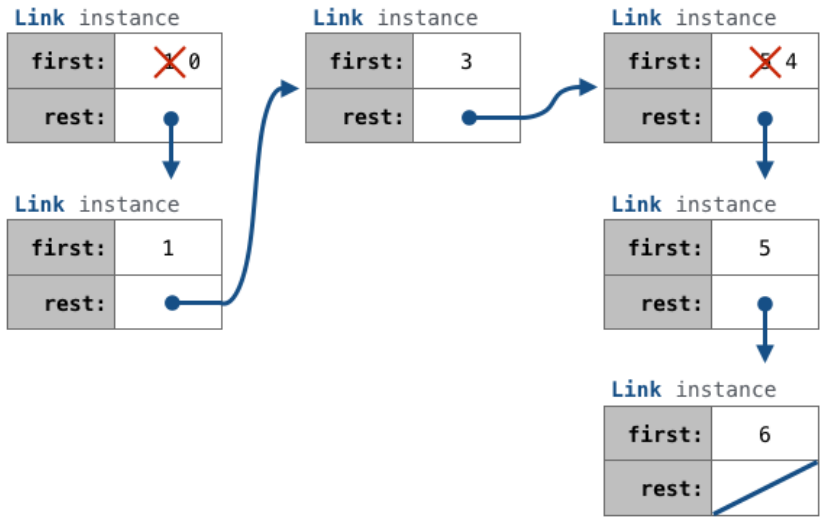
```

s:



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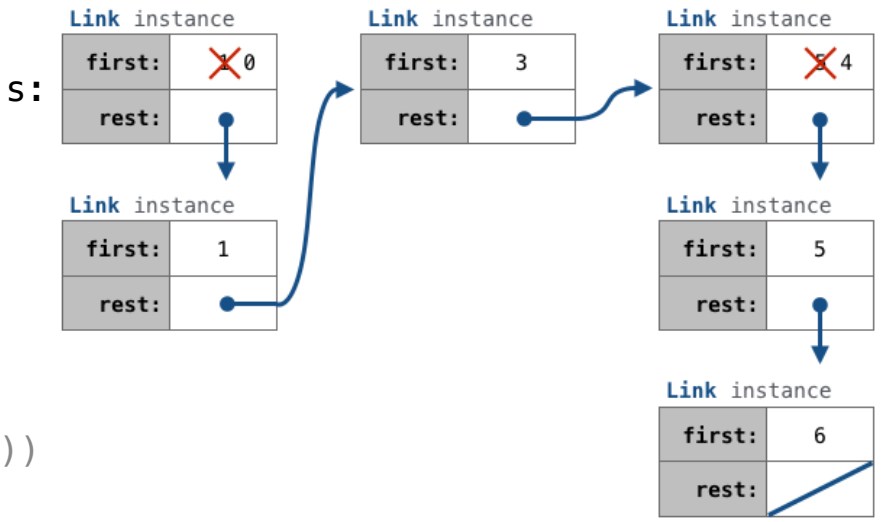

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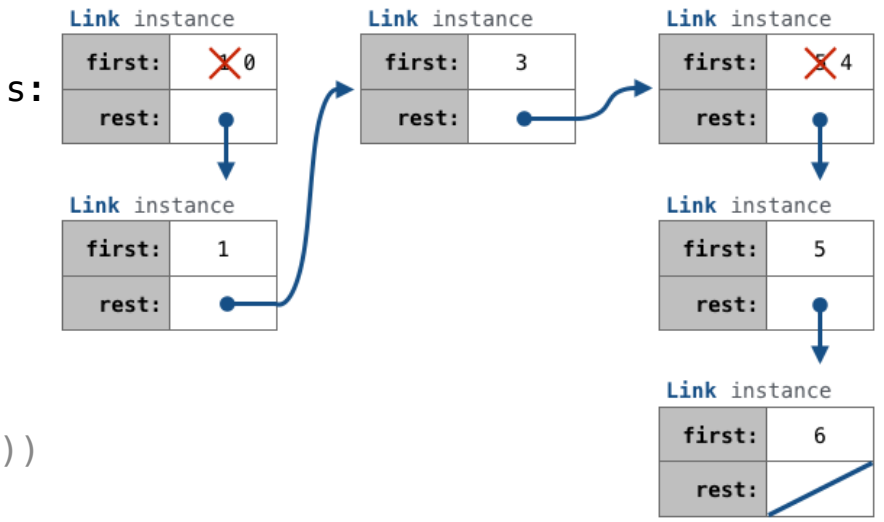

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```

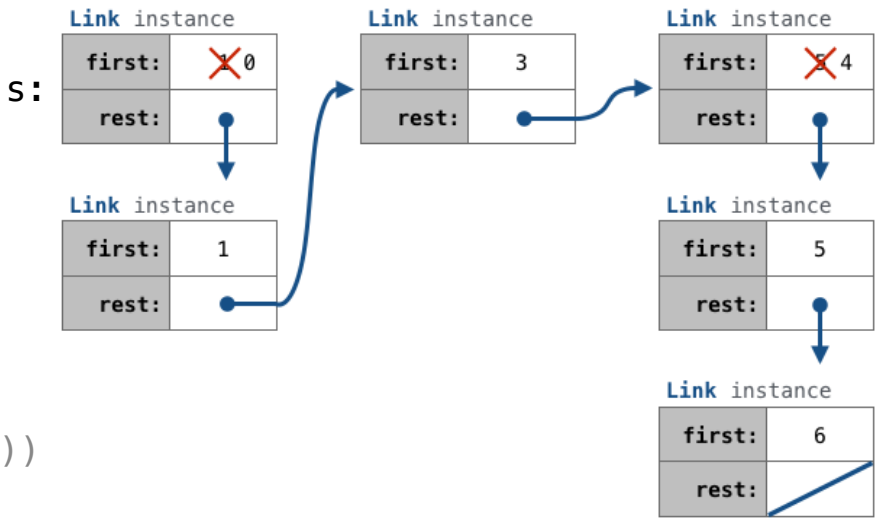


```
    if s.first > v:
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    elif s.first < v and empty(s.rest):
        s.rest = ____Link(v, s.rest)____
    elif s.first < v:
        ____add(s.rest, v)____
    return s
```

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